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1 $\int \sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos^2 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + 1 + \frac{\cos 4x}{2}\right)\left(1 - \cos 2x\right)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$$

$$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x * \cos 2x)$$

$$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2}\cos 4x \cos 2x)$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{32}\left[10 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$$

$$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

Let $\sin^6 x = R$

$$R = \int \frac{1}{32}(10 - 15\cos 2x + 6\cos 4x - \cos 6x)\,dx$$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

2 $\cos^4 x \sin^3 x$

$u = \cos x$ , $\frac{du}{dx} = -\sin x$ , $dx = \frac{-du}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x = \int \sin x \cdot \sin^2 x \, u^4 \frac{-du}{\sin x}$

$= -\int (1 - u^2) u^4 \, du$

$= -\int u^4 - u^6 \, du$

$= -\left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

$\int \cos^4 x \sin^3 x \; \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

3 $\int \cos x \sin 3x \, dx$

$\sin 3x = 3\cos^2 x \sin x - \sin^3 x$

$\sin^2(x) = 1 - \cos^2 x$

$= \int -(\cos x - 4\cos^3 x) \sin x \, dx$

$\cos x = 4$

$-\sin x\,dx \to du$

$= -\int (4u^3 - u)\,du$

$= \int (u - 4u^3)\,du$

$= \frac{u^2}{2} - u^4 + C$

$= \frac{\cos^2 x}{2} - \cos^4 x + C$