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Question

Integrate the following functions

1) $\sin^6 x$

Solution:- $\int \sin^6 x \, dx$

$\sin^6 x = (\sin^3 x)^2 \cdot \sin^2 x$

$= \int (\sin^2 x)^3 dx$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}(1 - 2\cos 2x + (\frac{1+\cos 4x}{2}))(1-\cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 4\cos^2 2x - \cos 4x \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x) - \frac{1}{2}(\cos 6x + \cos 2x))$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x))$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{(\cos 6x + \cos 2x)}{2})$

$= \frac{1}{16}\left(\frac{\cancel{6}-14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x}{2}\right)$

$= \frac{1}{32}(10 - 15\cos 2x + 6\cos 4x - \cos 6x)$.

$\therefore \int \sin^6 x\, dx = 1/32 \int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\, dx$

$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \overset{3}{\cancel{6}}\frac{\sin 4x}{\cancel{4}_2} - \frac{\sin 6x}{6}\right] + C$

$= \frac{5}{16}x - \frac{15\sin 2x}{64} + \frac{3\sin 4x}{64} - \frac{\sin 6x}{192} + C$

$= \frac{1}{192}(60x - 45\sin 2x + 9\sin 4x - \sin 6x) + C$

2) $\cos^4 x \sin^3 x$

Solution :- $\int \cos^4 x \sin^3 x\, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$; $dx = \frac{-du}{\sin x}$

$= \int u^4 . \cancel{\sin x} . \sin^2 x . \frac{-du}{\cancel{\sin x}}$

$= -\int u^4 . \sin^2 x . du$

$= -\int \sin^2 x . u^4 du$

$= -\int (1 - \cos^2 x) u^4 du$

$= -\int (1 - u^2) . u^4 du$

$= -\int (u^4 - u^6)\, du$

$= \int (-u^4 + u^6)\, du$

$= \int (u^6 - u^4)\, du$

$\therefore = \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

3) $\cos x \sin^3 x$

solution:- $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x, \therefore dx = \frac{du}{\cos x}$

$= \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$

$= \int u^3 du$

$= \frac{u^4}{4} + c$

$\therefore = \frac{\sin^4 x}{4} + c$