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Course: MAT 104
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Assignment

① $\int \sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1 - \cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} 2\cos 4x \cos 2x$

$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right)$

$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right)$

$= \frac{1}{32}\left(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x + \cos 2x\right)$

$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$

Let $\sin^6 x = R$

$R = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\, dx$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{198} + C$$

② $\cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$$\int \cos^4 x \sin^3 x\, dx = \int \cancel{\sin x} \cdot \sin^2 x\, u^4 \frac{du}{\cancel{\sin x}}$$

$$= -\int (1-u^2) u^4 du$$

$$= -\int u^4 - u^6\, du$$

$$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$\int \cos^4 x \sin^3 x\, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

③ $\cos x \sin^3 x$

$\int \cos(x) \sin^3(x)\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \int u^3 du$

Apply power rule

$\int u^n du = \frac{u^{n+1}}{n+1}$

$\int u^3 du = \frac{u^{3+1}}{3+1}$

$= \frac{u^4}{4}$

$= \frac{(\sin x)^4}{4}$

$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$