Maths 104 Assignment 28/5/20

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Level: 100

Integrate the following function

1 $\int \sin^6 x \, dx$

Solu

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$$\left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$\frac{1}{8}\left(1-2\cos 2x+\cos^2 2x\right)\left(1-\cos 2x\right)$$

$$=\frac{1}{8}\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)\left(1-\cos 2x\right)$$

Solve with LCM

$$\frac{1}{16}\left(2-4\cos 2x+1+\cos 4x\right)\left(1-\cos 2x\right)$$

$$\frac{1}{16}\left(3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 2x-\cos 4x\cos 2x\right.$$

$$\frac{1}{6}\left[3-7\cos 2x+\cos 4x+2\left(2\cos^2 2x\right)-\frac{1}{2}\cos 4x \cdot \cos 2x\right.$$

$$\frac{1}{16}\left[3-7\cos 2x+\cos 4x+2\left(1+\cos 4x\right)-\frac{1}{2}\left(\cos 6x+\cos 2x\right)\right]$$

$$\frac{1}{16}\left[3-7\cos 2x+\cos 4x+2+2\cos 4x-\frac{1}{2}\left(\cos 6x+\cos 2x\right)\right]$$

$$\frac{1}{32}\left[6-14\cos 2x+2\cos 4x+4+4\cos 4x-\cos 6x-\cos 2x\right]$$

$$=\frac{1}{32}\left[10-15\cos 2x+6\cos 4x-\cos 6x\right.$$

$$=\frac{1}{32}\int\left[10-15\cos 2x+6\cos 4x-\cos 6x\right]dx$$

$$=\frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{6\sin 4x}{4}-\frac{\sin 6x}{6}\right]+C$$

$$=\frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{3\sin 4x}{2}-\frac{\sin 6x}{6}\right]+C$$

$$\int \sin^6 x \, dx = \frac{1}{192}\left[60x-45\sin 2x+3\sin 4x-\sin 6x\right]+C$$

2 $\cos^4 x \sin^3 x$

Solu

Let $u = \cos x$

$\frac{du}{dx} = -\sin x \qquad dx = \frac{-du}{\sin x}$

$\sin^2 x = 1-\cos^2 x$

$$\int u^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$$

$$= -\int \left(1-\cos^2 x\right) \cdot u^4 \, du$$

$$= \int (U^2 - 1) \cdot U^4 \, du$$

$$= \int (U^6 - U^4) \, du$$

$$= \left[\frac{U^7}{7} \cancel{+} - \frac{U^5}{5} \right] + C$$

$$\int \cos^4 x \sin^3 x = \quad = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

3 $\quad \cos x \sin^3 x \, dx$

Solu

Let $U = \sin x$

$\frac{du}{dx} = \cos x \quad , \; du = \cos x \, dx$

$$\int U^3 du = \frac{U^{3+1}}{3+1} + c$$

$$= \frac{U^4}{4} + c$$

$$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$$