Adooh Nornubari Favour
19/MHS01/045
MBBS

$\sin^6 x\,dx = \left(1-\frac{\cos 2x}{2}\right)^2\left(1-\frac{\cos 2x}{2}\right)$

$= \frac{1}{8}(1-2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1-2\cos 2x + \left(1+\frac{\cos 4x}{2}\right)\right](1-\cos 2x)$

$= 1/16\,(2-4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= 1/16\,(3-4\cos 2x + \cos 4x)(1-\cos 2x)$

$= 1/16\,(3-4\cos x + \cos 4x - 3\cos 2x +$
$(4\cos^2 2x - \cos 4x\cos 2x)$

$= 1/16\,(3-7\cos 2x + \cos 4x + 4\cos^2 2x - \cos 4x\cos 2x)$

$= 1/16\,(3-7\cos 2x + \cos 4x + 2 \cancel{\times} 2\cos^2 2x)$

$- 1/2\,(\cos 6x + \cos 2x)$

$= 1/16\,(3-7\cos 2x + \cos 4x + 2(1+\cos 4x) -$
$1/2\,(\cos 6x + \cos 2x)$

$= 1/16\,\left(3-7\cos 2x + \cos 4x + 2 + 2\cos 4x - \left(\frac{\cos 6x}{2}\right.\right.$
$\left.\left. - \cos 2x/2\right)\right)$

$= 1/16\,\left(\frac{6-14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x}{2}\right)$

$= 1/32\,(10-15\cos 2x + 6\cos 4x - \cos 6x)$

$\int \sin^6 x\,dx = 1/32\int 10-15\cos 2x + 6\cos 4x - \cos 6x)\,dx$

$= 1/32\,\left(10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \sin 6x/6\right) + c$

$= 5/16\,x - 15\sin 2x/64 + \frac{3\sin 4x}{64} - \frac{\sin 6x}{192} + c$

$= 1/192\,(60x - 45\sin 2x + 9\sin 4x - \sin 6x) + c$

3 $\cos x \sin^3 x$

$\cancel{\cos x \sin x} = \int \cos x \sin^3 x \, dx$

$u = \cos x$

$du/dx = -\sin x$

$du = -\sin x \, dx$

$$\int \cos x \sin^3 x \, dx = \int \sin^3 x \cdot u \cdot -du/\sin x$$

$$= \int \cancel{\sin x} \cdot \sin^2 x \cdot u \cdot -\frac{du}{\cancel{\sin x}}$$

$$\cos x \sin^3 x = -\int \sin^2 x \cdot u \cdot du$$

$$= -\int (1-\cos^2 x) \cdot u\,du$$

$$= -\int (1-u^2) \cdot u\,du$$

$$= -\int (u-u^3)\,du$$

$$= \int (-u+u^3)\,du$$

$$\cos x \sin^3 x = \frac{u^4}{4} - \frac{u^2}{2} + C$$

$$\cos x \sin^3 x = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$$

///

2) $\cos^4 x \sin^3 x$

$\int \cos^4 x \sin^3 x \, dx$

when n is odd,

$u = \cos x$

$du/dx = -\sin x$, $dx = \dfrac{du}{-\sin x}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$= \int u^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \dfrac{-du}{\cancel{\sin x}}$

$= \int u^4 \cdot \sin^2 x \cdot -du$

$= -\int \sin^2 x \cdot u^4 du$

$= -\int (1 - \cos^2 x) \cdot u^4 du$

$= \int (u^2 - 1) \cdot u^4 du$

$= \int (u^6 - u^4) du$

$= \int \left(\dfrac{u^7}{7} - \dfrac{u^5}{5}\right) du + C$

$\cos^4 x \sin^3 x dx = \dfrac{(\cos x)^7}{7} - \dfrac{(\cos x)^5}{3} + C.$