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MATRIC NO: FI/MHS01/126

DEPARTMENT: MEDICINE AND SURGERY

COURSE: MATHEMATICS 102

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ASSIGNMENT.

Integrate the following functions.

1. $\sin^6 x$

2. $\cos^4 x \sin^3 x$

3. $\cos x \sin^3 x$

SOLUTION

1. $\int \sin^6 x$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= 1/8 (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= 1/8 \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1 - \cos 2x)$

$= 1/16 (1 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= 1/16 (3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$= 1/16 (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$= 1/16 (3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - 1/2 \cdot 2\cos 4x \cos 2x)$

$= 1/16 (3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - 1/2 (\cos 6x + \cos 2x)$

$= 1/16(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - 1/2(\cos 6x + \cos 2x)$

$= 1/32(8 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x + \cos 2x$

$= 1/32(10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

Let $\sin^6 x = P$

$P = 1/32 \int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

$P = 1/32 \left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right)$

$\therefore \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$

2. $\int \cos^4 x \sin^3 x$

Let $u = \cos x$; $du/dx = -\sin x$

$dx = -du/\sin x$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x = \int \cancel{\sin x} \cdot \sin x \; u^4 - du/\cancel{\sin x}$

$= -\int (1 - u^2) u^4 \, du$

$= -\int u^4 - u^6 \, du$

$= -(u^5/5 - u^7/7) + C$

$= u^7/7 - u^5/5 + C$

Recall $\rightarrow u = \cos x$

$$\Rightarrow \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$\therefore \int \cos^4 x \sin^3 x = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

3. $\int \cos x \sin^3 x$

Let $u = \sin x$; $du/dx = \cos x$

$$du = \cos x \, dx$$

$$\int \cos x \sin^3 x = \int u^3 du$$

$$\int u^3 du = \frac{u^{3+1}}{3+1} + C$$

$$\int u^3 du = \frac{u^4}{4} + C$$

Recall $u = \sin x$

$$\therefore \int \cos x \sin^3 x = \frac{\sin^4 x}{4} + C.$$