

$$3. \int \cos x \sin^3 x \, dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x \, dx$$

$$-du = \sin x \, dx$$

$$dx = \frac{-du}{\sin x}$$

$$\cos x \sin^3 x = u \cdot \sin x \cdot \sin^2 x$$

$$\int \cos x \sin^3 x \, dx = \int u \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$= -\int u \cdot \sin^2 x \cdot du$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\therefore \sin^2 x = 1 - u^2$$

$$\int \cos^4 x \sin^3 x \, dx = -\int u \cdot (1 - u^2) \cdot du$$

$$= -\int u - u^3 \, du$$

$$= -\left(\frac{u^2}{2} - \frac{u^4}{4}\right) + C$$

$$= \frac{u^4}{4} - \frac{u^2}{2} + C$$

recall $u = \cos x$

$$\int \cos^4 x \sin^3 x \, dx = \frac{(\cos x)^4}{4} - \frac{(\cos x)^2}{2} + C$$

$$\int \sin^6 x dx = \frac{1}{32} \int (10 - 15 \cos 2x + 6 \cos 4x - \cos 6x) dx$$

$$\int \sin^6 x = \frac{1}{32} \left(10x + \frac{15 \sin 2x}{2} + \frac{6 \cos 4x}{4} - \frac{\cos 6x}{6} \right) + C$$

$$\int \sin^6 x dx = \frac{10x}{32} - \frac{15 \sin 2x}{64} + \frac{6 \cos 4x}{128} - \frac{\cos 6x}{192} + C$$

2) $\int \cos^4 x \sin^3 x dx$

m is odd $\therefore u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\int \cos^4 x \sin^3 x dx = \int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$= -\int u^4 \cdot \sin^2 x \cdot du$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\therefore \sin^2 x = 1 - u^2$$

$$\int \cos^4 x \sin^3 x dx = -\int u^4 \cdot (1 - u^2) \cdot du$$

$$= -\int u^4 - u^6 du$$

$$= -\left(\frac{u^5}{5} - \frac{u^7}{7} \right) + C$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

recall $u = \cos x$

$$\int \cos^4 x \sin^3 x dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

LODOMARI IBITUN DOBURAII

MEDICINE AND SURGERY

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MAT 104

Integrate the following

but $\sin^2 x = \frac{1 - \cos 2x}{2}$

1) $\int \sin^6 x dx$

$$\sin^6 x dx = (\sin^2 x)^2 (\sin^2 x)$$

$$\therefore \int \sin^6 x dx = \int \left(\frac{1 - \cos 2x}{2} \right)^2 \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$\sin^6 x = \int \left(\frac{1 - 2\cos 2x + \cos^2 2x}{4} \right) \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{1}{8} (1 - 2\cos 2x + \cos^2 2x) (1 - \cos 2x)$$

$$= \frac{1}{8} \left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2} \right) (1 - \cos 2x)$$

$$= \frac{1}{16} (2 - 4\cos 2x + 1 + \cos 4x) (1 - \cos 2x)$$

$$= \frac{1}{16} (3 - 4\cos 2x + \cos 4x) (1 - \cos 2x)$$

$$= \frac{1}{16} (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$$

$$= \frac{1}{16} (3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} \cdot 2(\cos 4x \cos 2x))$$

$$= \frac{1}{16} (3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x))$$

$$= \frac{1}{16} (3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x))$$

$$= \frac{1}{32} (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$$

$$= \frac{1}{32} (6 + 4 - 14\cos 2x - \cos 2x + 2\cos 4x + 4\cos 4x - \cos 6x)$$

$$= \frac{1}{32} (10 - 15\cos 2x + 6\cos 4x - \cos 6x)$$