ITYOACHIMIN FAITH MNENA
19/MHS01/210
MBBS
MAT 104

1 $\sin^6 x$

Using the formula

$$\int \sin(x)^n dx = \frac{-1}{n}\sin(x)^{n-1}\cos(x) + \frac{n-1}{n}\int \sin(x)$$

$$\therefore \int \sin^6 x\,dx = \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\int \sin^4 x\,dx$$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{4}\int \sin^2 x\,dx\right)$$

Using $\sin^2 x = \frac{1-\cos 2x}{2}$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{4}\int \frac{1-\cos 2x}{2}\,dx\right)$$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{4}\cdot\frac{1}{2}\int (1-\cos 2x)\,dx\right)$$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{8}\int (1-\cos 2x)\,dx\right)$$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{8}\left(x - \frac{\sin 2x}{2}\right)\right)$$

$$= \frac{-1}{6}\sin^5 x\cos x + \frac{5}{6}\left(\frac{-1}{4}\sin^3 x\cos x + \frac{3}{8}x - \frac{3}{16}\sin 2x\right)$$

$$= -\frac{\sin^5 x\cos x}{6} - \frac{5\sin^3 x\cos x}{24} + \frac{5}{16}x - \frac{5\sin 2x}{32} + C$$

$$\therefore \int \sin^6 x\,dx = -\frac{\sin^3 x\cos x}{6} - \frac{5\sin^3 x\cos x}{24} + \frac{5}{16}x - \frac{5\sin 2x}{32} + C$$

2 $\int \cos^4 x \sin^3 x \, dx$

Since m is odd, $u = \cos x$, $\frac{du}{dx} = \sin x$, $dx = \frac{-du}{\sin x}$

$\therefore \int \cos^4 x \sin^3 x \, dx = \int u^4 \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$$= \int u^4 - \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$$

But $\sin^2 x = 1 - \cos^2 x$

$$= \int u^4 (1 - \cos^2 x) - du$$

$$= -\int u^4 (1 - u^2) \, du$$

$$= \int -u^4 (1 - u^2) \, du$$

$$= \int (-u^4 + u^6) \, du$$

$$= \frac{-u^5}{5} + \frac{u^7}{7} + c$$

$$\therefore \int \cos^4 x \sin^3 x \, dx = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$, $\frac{du}{dx} = \cos x$, $du = \cos x \, dx$, $dx = \frac{du}{\cos x}$

$$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$= \int u^3 \, du$$

$$= \frac{u^4}{4} + c$$

$$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$$