28TH MAY, 2020.

AKPOFURE TESE
19/MHS01/077
100 LEVEL
MEDICINE AND SURGERY
MEDICINE AND HEALTH SCIENCES
MAT 104 - GENERAL MATHEMATICS III (CALCULUS)

Integrate the following:

1. $\sin^6 x$

Solution

$\int \sin^6 x\,dx$.

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

Recall $\sin^2 x = \left(\frac{1-\cos 2x}{2}\right)$

$\therefore \sin^6 x = \left(\frac{1-\cos 2x}{2}\right)^2 \cdot \left(\frac{1-\cos 2x}{2}\right)$

$\sin^6 x = \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$\sin^6 x = \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1-\cos 2x\right)$

$\sin^6 x = \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$\sin^6 x = \frac{1}{16}\cdot(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$\sin^6 x = \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$\sin^6 x = \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2(\cos^2 2x) - 1/2 \cdot 2\cos 4x \cos 2x]$

$\sin^6 x = \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right)$

$$\sin^6 x = \frac{1}{16}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$$

$$\sin^6 x = \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

$$\int \sin^6 x\,dx = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\,dx$$

$$\int \sin^6 x\,dx = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right)$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + c$$

2. $\cos^4 x \sin^3 x$

Solution

$\int \cos^4 x \sin^3 x\,dx$

$m = \text{odd},\ n = \text{even} \ \therefore\ u = \cos x$

$du/dx = -\sin x \ ;\ dx = -du/\sin x$

$$\int \cos^4 x \sin^3 x\,dx = \int \cos^4 x \sin^2 x \sin x\,dx$$

$$\int \cos^4 x \sin^3 x\,dx = \int u^4 \cdot (1-\cos^2 x)\cancel{\sin x}\ \frac{-du}{\cancel{\sin x}}$$

$$\int \cos^4 x \sin^3 x\,dx = \int u^4 \cdot (1-\cos^2 x) \cdot -du$$

$$\int \cos^4 x \sin^3 x\,dx = -\int (1-u^2)u^4 \cdot du$$

$$\int \cos^4 x \sin^3 x\,dx = -\int (u^4 - u^6) \cdot du$$

$$\int \cos^4 x \sin^3 x\,dx = \int (-u^4 + u^6) \cdot du$$

$$\int \cos^4 x \sin^3 x\,dx = \frac{-u^5}{5} + \frac{u^7}{7}$$

$$\int \cos^4 x \sin^3 x\,dx = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\int \cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

3. $\cos x \sin^3 x$

Solution

$$\int \cos x \sin^3 x \, dx$$

$u = \sin x$

$$\frac{du}{dx} = \cos x, \quad dx = \frac{du}{\cos x}$$

$$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$\int \cos x \sin^3 x \, dx = \int u^3 \cdot du$$

$$\int \cos x \sin^3 x \, dx = \frac{u^{3+1}}{3+1} + C$$

$$\int \cos x \sin^3 x \, dx = \frac{u^4}{4} + C$$

$$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$$

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