Assignment

1) $\int \sin^6 x\,dx$

$$\int \left[\frac{1}{2}(1-\cos 2x)\right]^3 dx$$

$$\frac{1}{8}\int \left(1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x\right) dx$$

$$\frac{1}{8}\left(x - \tfrac{3}{2}\sin 2x + \tfrac{3}{2}x + \tfrac{3}{8}\sin 4x - \int (1-\sin^2 2x)\cos 2x\, d\right.$$

$$\frac{5}{16}x - \frac{3}{16}\sin 2x + \frac{3}{64}\sin 4x - \int (1-\sin^2 2x)\cos 2x$$

$$\frac{5}{16}x - \frac{3}{16}\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{16}\sin 2x + \frac{1}{48}\sin^3 2x + C$$

$\therefore$

$$\int \sin^6 x\,dx = \frac{5}{16}x - \frac{3}{16}\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{16}\sin 2x + \frac{1}{48}\sin^3 2x + C$$

2) $\int \cos^4(x)\sin^3(x)\,dx$

$$\sin^3(x)\cos^4(x) = \sin x \cdot \sin^2(x)\cdot\cos^4(x)$$
$$= \sin x\,[1-\cos^2(x)]\cos^4(x)$$

Let $u = \cos x$

$$\frac{du}{dx} = -\sin x \Rightarrow -du = \sin x\,dx$$

$$-1\int (1-u^2)\cdot u^4\,du$$

$$= -1\int u^4 - u^6\,du = -1\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$$

$$\therefore \int \cos^4(x)\sin^3(x)\,dx = -1\left[\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}\right] + C$$

3) $\int \cos x \sin^3 x \, dx$

$\sin x \cdot \sin^2 x \cdot \cos x$

$\sin x \left[1 - \cos^2 x\right] \cos x$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x \implies du = -\sin x \, dx$

$= -1 \int (1 - u^2) \cdot u \, du$

$= -1 \int u^2 - u^3 \, du$

$-1 \left[\frac{u^2}{2} - \frac{u^4}{4}\right] + C$

$\therefore \cos x \sin^3 x \, dx = -1 \left[\frac{\cos^2 x}{2} - \frac{\cos^4 x}{4}\right] + C$