05/2020 OBIADAZIE MICHEUE C.
MATH 104.
19/MHS01/272.

1: $\sin^6 x$ $\quad\quad$ $\sin^2 x = \frac{1-\cos 2x}{2}$

$= \int \sin^6 x\,dx$ $\quad\quad$ $\cos^2 x = \frac{1+\cos 2x}{2}$

$= \int (\sin^2 x)^3$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}\int (1-\cos 2x)(1-\cos 2x)(1-\cos 2x)$

$= \frac{1}{8}\int (1-\cos 2x)(1-2\cos 2x+\cos^2 2x)$

$= \frac{1}{8}\int (1-2\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x)$

$= \frac{1}{8}\int (1-3\cos 2x+3\cos^2 2x-\cos^3 2x)$

$= \frac{1}{8}\int 1-3\cos 2x+3\left[\frac{1+\cos 4x}{2}\right]-\cos^3 2x$

$[1-\sin^2 2x]$

$= \frac{1}{8}\int 1-3\cos 2x+\frac{3}{2}+\frac{3\cos 4x}{2}-\cos 2x+\cos 2x\sin^2 2x$

$= \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x$

$= \frac{1}{8}\left[\frac{5}{2}x - \frac{4\sin 2x}{2} + \frac{3\cos 4x}{8} + \frac{\sin^3 2x}{6}\right] + c$

$= \frac{5}{16} - \frac{4\sin 2x}{16} + \frac{3\cos 4x}{64} + \frac{\sin^3 2x}{48} + c$

2 $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \cancel{\sin x} \cdot \sin^3 x \cdot \frac{-du}{\cancel{\sin x}}$

$\int \cos^4 x \sin^3 x = -\int u^4 \cdot (1-\cos^2 x)$

$= -\int u^4 \cdot (1-u^2)$

$= -\int u^4 - u^6$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$\int \cos x \sin^3 x \, dx = \int u^3 \cdot du$$

$$\int \cos x \sin^3 x \, dx = \left[\frac{u^4}{4}\right] + c$$

$$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$$