

## MAT ASSIGNMENT

$$1 \int \sin^6 x$$

$$= (\sin^2 x)^3$$

$$\sin^2 x = \frac{1}{2} [1 - \cos 2x]$$

$$\sin^6 x = \left[ \frac{1 - \cos 2x}{2} \right]^3$$

$$\int \sin^6 x = \frac{1}{8} \int (1 - \cos 2x)^3$$

$$= \frac{1}{8} \int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x) dx$$

$$= \frac{1}{8} \int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x) dx$$

$$= \frac{1}{8} \int (1 - 3\cos 2x + 3(\frac{1 + \cos 4x}{2}) - \cos 2x(1 - \sin^2 2x)) dx$$

$$= \frac{1}{8} \int (1 - 3\cos 2x + \frac{3 + 3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 2x) dx$$

$$= \frac{1}{8} \int (\frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x) dx$$

$$= \frac{1}{8} \left[ \frac{5x}{2} - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6} \right] + C$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

$$2 \quad \int \cos^4 x \sin^3 x$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x dx, \quad dx = \frac{du}{-\sin x}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\int \sin x \cdot \sin^2 x \cdot \frac{u^4 du}{\sin x}$$

$$\int \cos^4 x \sin^3 x = \int \sin x (1 - \cos^2 x) u^4 du$$

$$\int \cos^4 x \sin^3 x = - \int (1 - u^2) u^4 du$$

$$\int \cos^4 x \sin^3 x = - \int (u^4 - u^6) du$$

$$\int \cos^4 x \sin^3 x = - \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C$$

$$\int \cos^4 x \sin^3 x = + \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

③  $\int \cos x \sin^3 x \, dx$  where both are odd

$$u = \sin x \Rightarrow \frac{du}{dx} = \cos x \quad dx = \frac{du}{\cos x}$$

$$\int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}} \Rightarrow \int u^3 \cdot du$$

$$\Rightarrow \frac{u^4}{4} + C \Rightarrow \text{where } u = \sin x$$

$$\Rightarrow \frac{\sin^4 x}{4} + C$$

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