① $\int \sin^6 x\,dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}\int (2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}\int (3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}\int (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos^2 2x * \cos 2x)$

$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2}\cos 4x \cos 2x\right.$

$\frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right.$

$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right.$

$= \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$

$= \frac{1}{32}\left[6 - 14\cos 2x + 6\cos 4x - \cos 6x\right]$

let $\sin^2 x = A$

$A = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

$= \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$

$\sin^2 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$

2. $\cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x$

$= \int \sin^2 x \cdot \sin x \times u^4 \cdot \frac{-du}{\sin x}$

$= -\int (1 - u^2) u^4 du$

$= -\int u^4 \cdot u^6 du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$

$= \frac{(\cos x)^5}{5} - \frac{(\cos x)^7}{7} + c$

$\cos^4 x \sin^3 x = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

③ $\cos x \sin^3 x$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$= \int u^3 \cdot du$

$= \left[\frac{u^4}{4}\right] + c$

$= \left[\frac{\sin x^4}{4}\right] + c$

$\int \cos x \sin^3 x \, dx$

$= \frac{(\sin x)^4}{4} + c$