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19/MHS01/185 MBBS

1. $\int \sin^6 x$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x$
$\cdot \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} 2\cos 4x \cos 2x$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}\cos 6x + \cos 2x$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x))$

$= \frac{1}{32}(10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

$I = \frac{1}{32}\int \cancel{\text{dx}} (10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

$I = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right)$

$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192}$

2. $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x = \sin x \cdot \sin^2 x \; u^4 \frac{-du}{\sin x}$

$= -\int (1-u^2)\, u^4 du = -\int u^4 - u^6 du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$

$I = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5}$ //

3 $\int \cos x \sin x^3 \, dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$= \int u^3 \cdot du$

$= \left[\frac{u^4}{4}\right] + C$

$= \left[\frac{\sin x}{4}\right]^4 + C$

$\int \cos x \sin x^3 = \frac{(\sin x)^4}{4} + C$