BABAKANO RABIYATU ALIYU

MBBS

19/mhs01/113

1) $\int \sin^6 x\,dx$

$\sin^6 x = (\sin^2 x)^2 \sin^2 x$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}\left(1-2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2-4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3-4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3-4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x\cos 2x)$

$= \frac{1}{16}\left(3-7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} \cdot 2\cos 4x\cos 2x\right)$

$= \frac{1}{16}\left(3-7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right)$

$= \frac{1}{32}\left[6-14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$

$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$

let $\sin 6x = R$

$R = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\,dx$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + c$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + c$$

2.) $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x$, $du/dx = -\sin x$, $dx = -du/\sin x$

$$\int \cos^4 + \sin^3 x \, dx = \int u^4 \cdot \sin x \cdot \sin^2 x \frac{du}{-\sin x} = -\int \cancel{u^4 \sin^2 x \, du}$$

$$\int (\cos^4 x \sin^3 x) dx = -\int u^4 (1 - \cos^2 x) \, du = -\int u^4 (1-u^2) \, du$$

$$= -\int (u^4 - u^6) \, du$$

$$\int (\cos^4 x \sin^3 x) dx = -\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + c = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\int (\cos^4 \sin^3 x) dx = \frac{\cos^7 x}{7} - \frac{\cos 5}{5} + c$$

3.) $\int (\cos x \sin^3 x) \, dx$

$$u + u = \sin x \rightarrow \frac{du}{dx} = \cos x \rightarrow du = \cos x \, dx$$

$\cancel{\int \cos x \sin^3}$

$$\int \cos x \sin^3 x \, dx = \int u^3 \, du = \frac{u^4}{4} + c = \frac{\sin^4 x}{4} + c$$