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19/MHS01/421
MAT 104 Assignment 11
MBBS

Integrate:

1) $\sin^6 x$

Solution

$$\int \sin^6 x \, dx = \int (\sin^2 x)^3 \, dx$$

$$\int (\sin^2 x)^3 \, dx = \int \left(\frac{1-2\cos 2x}{2}\right)^3 dx$$

$$\int \left(\frac{1-2\cos 2x}{2}\right)^3 dx = \frac{1}{8}\int (1-2\cos 2x)^3 = \frac{1}{8}\int (1-2\cos 2x)^2 (1-2\cos 2x) \, dx$$

$$\frac{1}{8}\int \left(1 - 6\cos 2x + 12\cos^2 2x - 8\cos^3 2x\right) dx$$

$$\frac{1}{8}\int \left[1 - 6\cos 2x + \cancel{12}^{6}\left(\frac{1+\cos 4x}{\cancel{2}}\right) - 8\cos^3 2x\right] dx$$

$$\frac{1}{8}\left[\int dx - 6\int \cos 2x \, dx + 6\int (1+\cos 4x) \, dx - 8\int \cos^3 2x \cdot dx\right]$$

Recall $\int \cos^3 2x \, dx = \int \cos^2 2x \cdot \cos 2x \, dx$

$$\int \cos^3 2x \, dx = \int \frac{\cos 2x(1+\cos 4x) \, dx}{2} = \frac{1}{2}\int (\cos 2x + \cos 2x \cos 4x) \, dx$$

But $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\therefore \cos 2x \cos 4x = \frac{1}{2}\left[\cos 6x + \cos 2x\right]$

$$\int \cos^3 2x \cdot dx = \frac{1}{2}\left\{\int \cos 2x \, dx + \frac{1}{4}\int \cos 6x \, dx + \frac{1}{4}\int \cos 2x \cdot dx\right\}$$

$$\int \cos^3 2x \cdot dx = \frac{1}{2}\left\{\frac{1}{2}\cdot\frac{\sin 2x}{2} + \frac{1}{4}\cdot\frac{\sin 6x}{6} + \frac{1}{4}\cdot\frac{\sin 2x}{2}\right\}$$

$$\int \cos^3 2x \cdot dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}$$

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$$\int \sin^6 x \, dx = \frac{1}{8}\left(\int dx - 6\int \cos 2x \, dx + 6\int (1+\cos 4x)\, dx - 8\int \cos^3 2x \cdot dx\right.$$

$$\frac{1}{8}x - \frac{3}{8}\sin 2x + \frac{3}{4}x + \frac{3}{16}\sin 4x - \frac{\sin 2x}{4} - \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + C$$

$$\int \sin 6x \cdot dx = \frac{7}{8}x - \frac{6\sin 2x}{8} + \frac{3}{16}\sin 4x - \frac{\sin 6x}{24} + C$$

2) $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x$ since m is odd

$\frac{du}{dx} = -\sin x$; $dx = \frac{-du}{\sin x}$

$$\int \cos^4 x \sin^3 x \, dx = \int \cos^4 x \sin x \cdot \sin^2 x \, dx$$

$$\int \cos^4 x \sin^3 x \cdot dx = \int \cos^4 x \sin x (1-\cos^2 x)\, dx$$

$$\int \cos^4 x \sin^3 x \cdot dx = \int u^4 \cancel{\sin x}\,(1-u^2)\cdot\left(\frac{-du}{\cancel{\sin x}}\right)$$

$$\int \cos^4 x \sin^3 x \cdot dx = -\int u^4 (1-u^2)\, du$$

$$\int \cos^4 x \sin^3 x \cdot dx = -\int (u^4 - u^6)\, du$$

$$\int \cos^4 x \sin^3 x \cdot dx = -\int\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + C$$

$$\int \cos^4 x \sin^3 x \cdot dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

3 $\int \cos x \sin^3 x \, dx$

$u = \cos x$ since m is odd

$\frac{du}{dx} = -\sin x$; $dx = \frac{-du}{\sin x}$

$$\int \cos x \sin^3 x \, dx = \int \cos x \cdot \cancel{\sin x}\,(1-\cos^2 x)\cdot\left(-\frac{du}{\cancel{\sin x}}\right)$$

$$\int \cos x \sin^3 x = -\int u(1-u^2)\cdot du$$

$$\int \cos x \sin^3 x = -\int (u - u^3)\, du = \int (u^3 - u^2)\, du = \frac{u^4}{4} - \frac{u^3}{3} + C$$

$$\int \cos x \sin^3 x = \frac{\cos^4 x}{4} - \frac{\cos^3 x}{3} + C$$