

RUFUS FORTUNE CHUNIAZE  
 19/mth301/388  
 MAT 104

1.  $\int \sin^6 x \, dx$

Using reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \int \sin^4 x \, dx$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left[ -\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x \, dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{24} \left[ \frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x \, dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5}{8} \left[ -\frac{1}{2} \sin x \cos x + \frac{1}{2} \int \sin^0 x \, dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$$

2.  $\int \cos^4 x \sin^3 x \, dx$

~~$\int uv = uv - \int v \, du$~~

let  $u = \cos x$  let

$du = -\sin x$

~~$\int \sin x \sin^2 x (\cos^2 x)^2 \, dx$~~

~~$\int \sin x (1 - \cos^2 x) (\cos^2 x)^2 \, dx$~~

$u = \cos x \quad du = -\sin x \, dx \quad dx = \frac{du}{-\sin x}$

~~$-\int \sin x (1 - \cos^2 x) (\cos^2 x)^2 \, du$~~

~~$-\int (1 - u^2)(u^2)^2 \, du$~~

~~$-\int (1 - u^2)(u^4) \, du$~~

~~$-\int (u^4 - u^6) \, du$~~

~~$= -\frac{u^5}{5} + \frac{u^7}{7} + C$~~

$= -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$

$$3. \int \cos x \sin^3 x \, dx$$

$$\int \cos x \sin x \sin^2 x \, dx$$

$$\int \cos x \sin x (1 - \cos^2 x) \, dx$$

$$\text{let } u = \cos x \quad du = -\sin x \, dx \quad dx = \frac{du}{-\sin x}$$

$$= - \int \sin x (1 - u^2) u \cdot \frac{du}{\sin x}$$

$$= - \int (1 - u^2) u \cdot du$$

$$= - \int (u - u^3) \cdot du$$

$$= - \frac{u^2}{2} + \frac{u^4}{4} + C$$

$$= - \frac{1}{2} \cos^2 x + \frac{1}{4} \cos^4 x + C$$