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MATHS 104
MEDICINE & SURGERY

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$$\int \sin^6 x\,dx = \int \sin^2 x \cdot \sin^4 x$$

$$= \int \left(\frac{1-\cos 2x}{2}\right) \cdot \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \int \frac{1-\cos 2x}{2} \cdot \frac{1-\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}$$

$$= \frac{1}{8}\int (1-\cos 2x)(1-2\cos 2x+\cos^2 2x)$$

$$= \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x-\cos^3 2x$$

$$= \frac{1}{8}\int 1-3\cos 2x+3\left[\frac{1+\cos 4x}{2}\right]-\cos 2x\left(1-\cancel{\sin^2 2x}\ \sin^2 2x\right]$$

$$= \frac{1}{8}\int 1-3\cos 2x+\frac{3}{2}+\frac{3\cos 4x}{2}-\cos 2x+\cos 2x\sin^2 2x$$

$$= \frac{1}{8}\int \frac{5}{2}-4\cos 2x+\frac{3\cos 4x}{2}+\cos 2x\sin^2 2x$$

$$= \frac{1}{8}\left[\frac{5x}{2}-\frac{4\sin 2x}{2}+\frac{3\cos 4x}{8}+\frac{\sin^3 2x}{6}\right]+c$$

$$\int \sin^6 x\,dx = \frac{1}{8}\left[\frac{5x}{2}-2\sin 2x+\frac{3\cos 4x}{8}+\frac{\sin^3 2x}{6}\right]+c$$

3 $\int \cos x \sin^3 x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int \cos x \sin^3 x = -\int u \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{du}{\cancel{\sin x}}$

$\int \cos x \sin^3 x = -\int \sin^2 x \cdot u\,du$

$\int \cos x \sin^3 x = -\int (1-\cos^2 x)\, u\,du$

$\int \cos x \sin^3 x = \int (u^2-1)\, u\,du$

$\int \cos x \sin^3 x = \int (u^3-u)\,du$

$\int \cos x \sin^3 x = \left[\frac{u^4}{4} - \frac{u^2}{2}\right]$

$\therefore \int \cos x \sin^3 x = \frac{(\cos x)^4}{4} - \frac{(\cos x)^2}{2} + c$

2 $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

since $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x = -\int \sin^2 x \cdot u^4 du$

$\int \cos^4 x \sin^3 x = -\int (1 - \cos^2 x) u^4 du$

$\therefore \int \cos^4 x \sin^3 x = \int (u^2 - 1) u^4 du$

$\int \cos^4 x \sin^3 x = \int (u^6 - u^4) du$

$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + c$

$\therefore \int \cos^4 x \sin^3 x = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$