ORKAA SAMUEL TERUNGWA
19/MHS01/358
MATHS ASSIGNMENT

1. $\sin^6 x$

Solution.

$$\int \sin^6 x \, dx = \int \sin^2 x \cdot \sin^4 x$$

$$= \int \left(\frac{1-\cos 2x}{2}\right) \cdot \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \int \frac{1-\cos 2x}{2} \cdot \frac{1-\cos^2 x}{2} \cdot \frac{1-\cos^2 x}{2}$$

$$= \frac{1}{8}\int (1-\cos 2x)(1-\cos 2x)(1-\cos 2x)$$

$$= \frac{1}{8}\int (1-\cos 2x)(1-2\cos 2x + \cos^2 x)$$

$$= \frac{1}{8}\int (1-3\cos 2x + 3\cos^2 2x - \cos^3 2x$$

$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \cos 2x\,(1-\sin^2 2x$$

$$= \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \cos 2x - \cos 2x \sin^2 2x$$

$$= \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x$$

$$= \frac{1}{8}\left[\frac{5x}{2} - \frac{4\sin 2x}{2} + \frac{3\cos 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$$

$$= \frac{1}{8}\left[\frac{5}{2}x - 2\sin 2x + \frac{3\cos 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$$

2. $\int \cos^4 x \sin^3 x$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x \, dx = \int u^4 \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$

$= -\int u^4 \cdot (1 - \cos^2 x)$

$= -\int u^4 \cdot (1 - u^2)$

$= -\int u^4 - u^6$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7} \right] + C$

It now becomes,

$\frac{u^7}{7} - \frac{u^5}{5} + C$

Substituting $u = \cos x$ we have,

$\int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3. $\int \cos x \sin^3 dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\therefore dx = \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$

$= \int u^3 du = \left[\frac{u^4}{4} \right] + C \Rightarrow \frac{\sin^4 x}{4} + C$