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(1) $\int \sin^6 x \, dx$

$= \int \sin^2 x \, (\sin^2 x)^2 \, dx$

Recall

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\sin^4 x = \left(\frac{1 - \cos 2x}{2}\right)^2$

$= \int \frac{1 - \cos 2x}{2} \left(\frac{1 - \cos 2x}{2}\right)^2 dx$

$= \frac{1}{8} \int 1 - \cos 2x \, (1 - \cos 2x)^2 \, dx$

$= \frac{1}{8} \int 1 - \cos 2x \, (1 - 2\cos 2x + \cos^2 2x) \, dx$

$= \frac{1}{8} \int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\cos^2 2x = \frac{1 + \cos 4x}{2}$

$$\cos^3 2x = \cos 2x\left(\frac{1+\cos 4x}{2}\right) = \frac{\cos 2x}{2} + \frac{1}{2} + \frac{\cos 16x}{2}$$

$$= \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \frac{\cos 2x}{2} - \frac{1}{2} - \frac{\cos 16x}{2}\,dx$$

$$= \frac{1}{8}\int \frac{4}{2} - 3\cos 2x + \frac{3\cos 4x}{2} - \frac{\cos 2x}{2} - \frac{\cos 16x}{2}\,dx$$

$$= \frac{1}{8}\left[2x - \frac{3\cos 2x}{2} + \frac{3\cos 4x}{8} - \frac{\cos 2x}{4} - \frac{\cos 16x}{32}\right] + c$$

$$= \frac{2x}{8} - \frac{3\cos 2x}{16} + \frac{3\cos 4x}{64} - \frac{\cos 2x}{32} - \frac{\cos 16x}{256} + c$$

$$= \frac{1}{4}x - \frac{3\cos 2x}{16} + \frac{3\cos 4x}{64} - \frac{\cos 2x}{32} - \frac{\cos 16x}{256} + c$$

2) $\int \cos^4 x \sin^3 x \, dx$

since n is odd

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cancel{\cos x} (\cos^3 x) \times u^3 x \frac{du}{\cancel{\cos x}}$

$\int \cos x (\cos^2 x) u^3 du$

$\cos^2 x = 1 - \sin^2 x$

$\int \cos x (1 - \sin^2 x) u^3 du$

$\int \cos x - \cos x \sin^2 x \; u^3 du$

$\int \cos^4 x \sin^3 x \, dx = \sin x - \sin^3 x$

$\int \cos x (1 - u^2) u^3 du$

$\int \cos x (u^3 - u^5) du$

$= \sin x \left[\frac{u^4}{4} - \frac{u^6}{6} \right] + c$

$= \sin x \left[\frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} \right] + c$

(3) $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cancel{\cos x} \cdot u^3 \frac{du}{\cancel{\cos x}}$

$\int u^3 \, du$

$= \frac{u^4}{4} + C$

$\int \cos x \sin^3 x \, dx = \frac{(\sin x)^4}{4} + C$

$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$