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19/MHS01/351 (MBBS)

1. $\int \sin^6 x\, dx$

$$\int \sin^6 x\, dx = (\sin^2 x)^2 (\sin^2 x)$$

$$\int \sin^6 x\, dx = \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$\int \sin^6 x\, dx = \frac{1}{8}\int \left(1 - 2\cos 2x + \cos^2 2x\right)\left(1 - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{8}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1 - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{16}\int \left(2 - 4\cos 2x + 1 + \cos 4x\right)\left(1 - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{16}\int \left(3 - 4\cos 2x + \cos 4x\right)\left(1 - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{16}\int \left(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{16}\int \left(3 - 7\cos 2x + \cos 4x + 2\left(2\cos^2 2x\right) - \frac{1}{2}\, 2\cos 4x \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{32}\int \left(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{32}\int \left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right) dx$$

$$\int \sin^6 x\, dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right] + c$$

$$\int \sin^6 x\, dx = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + c$$

$\overline{\quad 8} \quad \overline{\quad 4} \quad - \quad \overline{\quad 64}$

2 $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad ; \; dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x = \int \cos^4 x \cdot \sin x \sin^2 x \, dx$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \sin x \sin^2 x \cdot \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \cancel{\sin x} \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \sin^2 x \cdot -du$

$\int \cos^4 x \sin^3 x = \int \sin^2 x \cdot u^4 \cdot -du$

$\cancel{\int \cos^4 x} \quad \sin^2 x = 1 - \cos^2 x$

$\therefore \int \cos^4 x \sin^3 x = \int (1 - \cos^2 x) \, u^4 \cdot -du$

$\int \cos^4 x \sin^3 x = \int (1 - u^2) \, u^4 \cdot -du$

$\int \cos^4 x \sin^3 x = \int (u^4 - u^6) \cdot -du$

$\int \cos^4 x \sin^3 x = \int u^6 - u^4 \, du$

$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5} \right] + c$

$\int \cos^4 x \sin^3 x = \frac{u^7}{7} - \frac{u^5}{5} + c$

where $u = \cos x$

$\therefore \int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\therefore \int \cos x \sin^3 x = \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$\int \cos x \sin^3 x = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$

$\int \cos x \sin^3 x = \int u^3 \, du$

$\int \cos x \sin^3 x = \frac{u^4}{4} + c$

where $u = \sin x$

$\therefore \int \cos x \sin^3 x = \frac{\sin^4 x}{4} + c$