

1. $\int \sin^6 x$

using Reduction formula

$$u = \sin x \Rightarrow du = \cos x dx$$

$$S_n = \frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} S_{n-2}$$

$$S_6 = \frac{-\sin^5 x \cos x}{6} + \frac{6-1}{6} S_{6-2}$$

$$S_6 = \frac{\sin^5 x \cos x}{6} + \frac{5}{6} S_4$$

obtain expression for S_4

$$S_4 = \frac{-\sin^3 x \cos x}{4} + \frac{3}{4} S_2$$

$$S_2 = \frac{-\sin x \cos x}{2} + \frac{1}{2} S_0$$

$$S_0 = \int \sin^0 x dx = \int 1 dx = x$$

So into S_2 , then S_2 into S_4

$$S_4 = \frac{-\sin^3 x \cos x}{4} + \frac{3}{4} \left(\frac{-\sin x \cos x}{2} + \frac{1}{2} x \right)$$

$$S_4 = \frac{-\sin^3 x \cos x}{4} - \frac{3 \sin x \cos x}{4} + \frac{3}{8} x$$

S_4 into S_6

$$S_6 = \frac{-\sin^5 x \cos x}{6} + \frac{5}{6} \left(\frac{-\sin^3 x \cos x}{4} - \frac{3 \sin x \cos x}{4} + \frac{3}{8} x \right)$$

$$S_6 = \frac{-\sin^5 x \cos x}{6} - \frac{5 \sin^3 x \cos x}{24} - \frac{5 \sin x \cos x}{16} + \frac{5}{16} x$$

$$2 \cos^4 x \sin^3 x$$

$$\cos^4 x \sin^3 x = \int \sin^2 x \cos^4 x \sin x dx$$

$$u = \sin x \Rightarrow du \Rightarrow \cos x dx$$

$$\int \cos^4 x \sin^3 x = \int u^3$$

$$\text{let } u = \cos x \Rightarrow du \Rightarrow -\sin x dx \Rightarrow -du = \sin x dx$$

$$\text{Also, } \sin^2 x = 1 - \cos^2 x$$

$$= 1 - u^2$$

$$\text{as } \cos^4 x = u^4$$

Substituting, we have

$$-\int (1 - u^2) u^4 du = \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$$

$$3 \cos x \sin^3 x$$

$$u = \sin x \Rightarrow du \Rightarrow \cos x dx$$

$$\int \cos x \sin^3 x dx = \int du u^3 du = \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} \sin^4 x + C$$