Name : Anyakwo Emmamuella Chineze.

Dept : MBBS

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Ans

$n^6x \Rightarrow \int \sin^2 x (\sin^2 x)^2$

$= \left[\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)^2$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)$

$= \frac{1}{8}\int 1-2\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x\,dx$

$= \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x-\cos^3 2x\,dx$

$= \frac{1}{8}\int 1-3\cos 2x+3\left[\frac{1+\cos 4x}{2}\right]-\cos^3 2x$

$\int \cos^3 2x = \cos 2x \cdot \cos^2 2x\,dx$

$= \cos 2x(1-\sin^2 2x)$

$u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x \qquad dx = \frac{du}{2\cos 2x}$

$\frac{1}{2}\int \cos 2x - \cancel{\cos 2x} \cdot \sin^2 2x \cdot \frac{du}{2\cancel{\cos 2x}}$

$= \frac{1}{2}\int \cos 2x - \int u^2\,du$

$= \frac{\sin 2x}{4} - \frac{\sin^3 2x}{6} + c$

$= \frac{1}{1}\int 1-3\cos 2x+3\left[\frac{1+\cos 4x}{2}\right]-\frac{1}{2}\left[\cos 2x-u^2\,du.\right.$

ii) $\cos^4 x \sin^3 x \rightarrow \int \cos^4 x \sin^3 x \rightarrow \int \sin^3 x \cos^4 x$

Solution:

$u = \cos x$

$\frac{du}{dx} = \sin x \rightarrow dx = -du/\sin x$

Recall: $\sin^2 + \cos^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$\int \sin x \cdot \sin^2 x \cdot u^4 \cdot \frac{-du}{\sin x}$

$-\int \sin^2 x \cdot u^4 du$

$-\int (1-\cos^2 x) \cdot u^4 du$

Recall $u = \cos x$

$-\int (1-u^2) \cdot u^4 du$

$\int (u^2-1) \cdot u^4 du$

$\int (u^6-u^4) du$

$\frac{u^7}{7} - \frac{u^5}{5} + c \qquad \therefore \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c.$

iii) $\cos x \sin^3 x \qquad \therefore u = \sin x$

$du/dx = \cos x$

$\cos x \cdot \sin x \cdot \sin^2 x \qquad dx = du/\cos x$

$\int \cos x \sin^3 x \, dx =$

$\cancel{\cos x} \sin^3 x \frac{du}{\cancel{\cos x}}$

$\Rightarrow u^3 du$

$= \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$