

$$\textcircled{1} \quad \sin^6 x$$

Using reduction formula

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \sin^6 x \, dx = -\frac{\sin^5 x \cos x}{6} + \frac{5}{6} \int \sin^4 x \, dx$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left( -\frac{\sin^3 x \cos x}{4} + \frac{3}{4} \int \sin^2 x \, dx \right)$$

$$\Rightarrow \frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x +$$

$$\frac{5}{8} \left( -\frac{\sin x \cos x}{2} + \frac{1}{2} \int dx \right)$$

$$\Rightarrow \frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$$

$$\textcircled{2} \int \cos^4 x \sin^5 x \, dx$$

Soln

$$\int \underbrace{\cos^4 x \cdot \sin^2 x}_{f(u)} \underbrace{\sin x \, dx}_{du}$$

$$\Rightarrow \int (1 - \cos^2 x) \cos^4 x \cdot -\sin x \, dx$$

$$\text{Let } u = \cos x$$

$$du = -\sin x \, dx$$

$$= -1 \int (1 - u^2) u^4 \, du$$

$$= -1 \int u^4 - u^6 \, du = -1 \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C$$

$$\Rightarrow -1 \left( \frac{\cos^5 x}{5} - \frac{\cos^7 x}{7} \right) + C$$

$$\textcircled{3} \int \cos x \sin^3 x \, dx$$

Soln

$$u = \sin x$$

$$du = \cos x$$

$$\int u^3 du = \frac{u^4}{4} = \frac{\sin^4 x}{4} + C$$