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MAT NO: 19/MHS01/274

Integrate the following functions

1. $\sin^6 x$
2. $\cos^4 x \sin^3 x$
3. $\cos x \sin^3 x$

Solution

1. $\int \sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}\int (2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}\int (3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}\int 3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \times \cos 2x$

$= \frac{1}{16}\int 3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2}(2\cos 4x \times \cos 2x)$

$= \frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \times (2\cos 4x \cos 2x)$

$= \frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)$

$= \frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)$

$$= \frac{1}{32}\int (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$$

$$= \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\, dx$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$$

$$\int \sin^6 x\, dx = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{3\sin 4x}{64} - \frac{\sin 6x}{192} + C$$

2. $\int \cos^4 x \sin^3 x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

Recall that $\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$\int \cos^4 x \sin^3 x \, dx = \int u^4 \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$

$= \int u^4 \sin^2 x - du \quad = -\int u^4 \sin^2 x \, du$

$= -\int \sin^2 x \cdot u^4 du$

$= -\int (1 - \cos^2 x) \cdot u^4 du$

$= -\int (1 - u^2) u^4 du$

$= -\int (u^4 - u^6) du$

$= \int (u^6 - u^4) du$

$= \left[\frac{u^7}{7} - \frac{u^5}{5} \right] + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3. $\int \cos x \sin^3 x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$\int \cos x \sin^3 x \, dx = \int u^3 du$

$= \int u^3 du$

$= \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$