MAT101 MBBS

Integrate the following function

1) $\sin^6 x$

$= \int \sin^2 x \, (\sin^2 x)^2$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x + \cos^2 2x}{4}\right)$

$= \frac{1}{8}\int (1 - 2\cos 2x + \cos^2(2x) - \cos(2x) + 2\cos^2(2x) - \cos^3 2x \, dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \cos^3(2x)$ ......(i)

$\int \cos^3 2x = \cos 2x \cdot \cos^2(2x) \, dx$

$= \cos 2x(1 - \sin^2 2x)$

Let $u = \sin 2x$, $du/dx = 2\cos 2x$; $dx = du/2\cos 2x$

$\frac{1}{2}\int \cos 2x - \left[\cos 2x \cdot \sin^2(2x) \cdot \frac{du}{2\cos 2x}\right]$

$= \frac{1}{2}\int \cos 2x - \int u^2 \, du$

$= \frac{1}{2}\left[\frac{\sin 2x}{2} - \frac{u^3}{3}\right] + c$

$= \frac{\sin 2x}{4} - \frac{\sin^3(2x)}{6} + c$

From equation(i),

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \frac{1}{2}\left(\frac{\cos 2x}{8} - \sin^2(2x)\right)$

$\int \sin^4 x = (\sin^2 x)^2 = (1-\cos^2 x)^2$

$= (1 - 2\cos^2 x + \cos^4 x)$

2)

$8[(2\ )(\ 2\ )(\ 8\ )(\ 4\ )(\ 6\ )]$

$= \frac{5x}{16} - \frac{3\sin 2x}{16} + \frac{3\sin 4x}{64} - \frac{\sin 2x}{32} + \frac{\sin^3 2x}{48}) + C$

2) $\cos^4 x \sin^3 x$

$= \int \cos^4 x \sin^3 x = \int \sin^3 x \cos^4 x$

m is odd

$\therefore u = \cos x$

$\frac{du}{dx} = -\sin x \ ; \ dx = \frac{-du}{\sin x}$

recall, $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$

$\int \sin x \cdot \sin^2 x \cdot u^4 \cdot \left(-\frac{du}{\sin x}\right)$

$-\int \sin^2 x \cdot u^4 \, du$

$-\int (1-\cos^2 x) \cdot u^4 \, du$

$-\int (1-u^2) \cdot u^4 \, du$

$-\int (u^4 - u^6) \, du$

$\int (u^6 - u^4) \, du \ \cancel{= \int u^4 \, du}$

$= \int u^6 - u^4$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3) $\cos x \sin^3 x$

$= \cos x \ \sin x \ \sin^2 x$

$u = \sin x \ ; \ \frac{du}{dx} = \cos x, \ dx = \frac{du}{\cos x}$

$\int \cos x \cdot \sin^2 x \cdot u \cdot \frac{du}{\cos x} = \int \sin^2 x \cdot u \, du$

$= \int u^2 u \, dx = \int u^3 dx$