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Matric No: 19/MHS01/104
Course Code: Math 104

Assignment

1) $\int \sin^6 x \, dx$

Solution

$$\int \sin^2 x \, (\sin^2 x)^2$$

$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)^2$$

$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right)$$

$$= \frac{1}{8}\int 1 - 2\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 x - \cos^3 2x \, dx$$

$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$$

$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left[\frac{1 + \cos 4x}{2}\right] - \cos^3 2x$$

$$\int \cos^3 2x = \cos 2x \cdot \cos^2 2x \, dx$$

$$= \cos 2x \, (1 - \sin^2 2x)$$

$U = \sin 2x \quad \frac{du}{dx} = 2\cos 2x$

$$dx = \frac{du}{2\cos 2x}$$

2) $\int \cos^4 x \sin^3 x \, dx$

Solution

m is odd $\therefore$ $u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$

recall : $\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$-u^2 du$

$\int \cancel{\sin x} \cdot \sin^2 x \cdot u^4 \cdot \frac{-du}{\cancel{\sin x}}$

$-\int \sin^2 x \cdot u^4 \, du$

$-\int (1 - \cos^2 x) \cdot u^4 \, du$

recall $u = \cos x$

$-\int (1 - u^2) \cdot u^4 \, du$

$\int (u^2 - 1) \cdot u^4 \, du$

$\int (u^6 - u^4) \, du$

$\frac{u^7}{7} - \frac{u^5}{5} + c$

$\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3) $\cos x \sin^3 x$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\cos x \cdot \sin x \cdot \sin^2 x$

$\int \cos x \sin^3 x \, dx =$

$\cos x \sin^3 x \cdot \frac{du}{\cos x}$

$= u^3 du$

$= \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$