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Dept: Medicine And Surgery
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Course: MAT 104

Assignment

Integrate the following functions;

1 $\sin^6 x$

2 $\cos^4 x \sin^3 x$

3 $\cos x \sin^3 x$

Solution

1 $\sin^6 x$

$\int \sin^6 x \, dx$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1 - \cos 2x}{2}\right)^2 \left(\frac{1 - \cos 2x}{2}\right)$$

$$= \frac{1}{3}(1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$$

$$= \frac{1}{3}\left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right)(1 - \cos 2x)$$

$$= \frac{1}{16}\left(\frac{2 - 4\cos 2x + 1 + \cos 4x}{2}\right)(1 - \cos 2x)$$

$$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$$

$$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x$$

$$= \frac{1}{16}(3 - 7\cos 2x - \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} 2\cos 4x \cos 2x \ldots$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x \ldots\right.$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$$

$$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

Let $\sin^6 x = R$

$$R = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\, dx$$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

2 $\int \cos^4 x \sin^3 x$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

Recall that $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$\int \cos^4 x \sin^3 x\, dx = \int u^4 \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$= \int u^4 \sin^2 x \, -du = -\int u^4 \sin^2 x\, du$$

$$= -\int \sin^2 x \cdot u^4\, du$$

$$= -\int (1 - \cos^2 x)\, u^4\, du$$

$$= -\int (1 - u^2)\, u^4\, du$$

$$= -\int (u^4 - u^6)\, du$$

$$= \int (u^6 - u^4)\, du$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$$

$$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

3 $\int \cos x \sin^3 x\, dx$

Let $u = \sin x$, $du/dx = \cos x$

$dx = \cos x\, dx$

$$\int \cos x \sin^3 x\, dx = \int u^3\, du$$

$$= \int u^3\, du$$

$$= \frac{u^4}{4} + C$$

$$\int \cos x \sin^3 x\, dx = \frac{\sin^4 x}{4} + C$$