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COURSE CODE: MAT 104

ASSIGNMENT

Integrate the following functions.

1) $\sin^6 x$

Solution

$\int \sin^6 x\,dx$

$\int \sin^6 x = \int \sin^2 x \cdot \sin^4 x\,dx$

$\int \sin^6 x = \int \frac{1-\cos 2x}{2} \cdot \frac{1-2\cos 2x+\cos^2 2x}{4}\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x - \cos^3 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \cos 2x(1-\sin^2 x)\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 x\,dx$

$\int \sin^6 x = \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\left[\frac{5}{2}x - \frac{4\sin 2x}{2} + \frac{3\sin 4x}{2} \times \frac{1}{4} + \int \cos 2x \sin^2 2x\,dx\right.$

$\int \cos 2x \sin^2 2x \, dx = \int \cancel{\cos 2x} (u)^2 \frac{du}{2\cancel{\cos 2x}}$

$\boxed{\begin{aligned} &\text{Let } u = \sin 2x \\ &\frac{du}{dx} = 2\cos 2x \\ &\therefore dx = \frac{du}{2\cos 2x} \end{aligned}}$

$\int \cos 2x \sin^2 2x \, dx = \int u^2 \frac{du}{2}$

$\int \cos 2x \sin^2 2x \, dx = \frac{1}{2} \int u^2 du$

$\int \cos 2x \sin^2 2x \, dx = \frac{1}{2}\left(\frac{u^3}{3} + C\right)$

$\int \cos 2x \sin^2 2x \, dx = \frac{1}{2}\left(\frac{\sin^3 2x}{3} + C\right)$

$\therefore \int \sin^6 x = \frac{1}{8}\left[\frac{5x}{2} - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$ //

2. $\cos^4 x \sin^3 x$

Solution

$\cos^4 x \sin^3 x = \int \cos^4 x \sin^2 x \sin x \, dx$

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$\cos^4 x \sin^3 x = \int \cos^4 x (1 - \cos^2 x) \sin x \, dx$

Let $u = \cos x$

$du = -\sin x \, dx$

$\cos^4 x \sin^3 x = -\int u^4 (1 - u^2) du$

$\cos^4 x \sin^3 x = -\int u^4 - u^6 \cdot du$

$\cos^4 x \sin^3 x = -\frac{u^5}{5} + \frac{u^7}{7} + C$

$\cos^4 x \sin^3 x = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

3 $\cos x \sin^3 x$

Solution

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x}\, u^3 \cdot \frac{du}{\cancel{\cos x}}$

$\therefore \int \cos x \sin^3 x \, dx = \int u^3 du$

$\int \cos x \sin^3 x \, dx = \frac{u^4}{4} + C$

$\therefore \int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C.$