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① $\sin^6 x$

$\int \sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x]$

$= \frac{1}{16}[3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x]$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \times 2\cos 4x \cos 2x]$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)]$

$= \frac{1}{16}[3 - 7\cos 2x - \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x]$

$= \frac{1}{32}[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$

$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$

$$\int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4} + c$$

$$\frac{1}{32}\int [10 - 15\cos 2x + 6\cos 4x - \cos 6x]\,dx$$

$$\frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$$

$$\frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + c$$

$$\frac{1}{192}\left[60x - 45\sin 2x + 9\sin 4x - \sin 6x\right] + c$$

2

$\cos^4 x \sin^3 x$

$\int \cos^4 x \sin^3 x \, dx$

$\int \sin^3 x \cos^4 x \, dx$

$$\sin^3 x \cos^4 x = (1-\cos^2 x)\cos^4 x \sin x$$
$$= \cos^4 x \sin x - \cos^6 x \sin x$$
$$= \int (\cos^4 x \sin x - \cos^6 x \sin x)\,dx$$
$$= \frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$$

3

$\cos x \sin^3 x$

$\int \cos x \sin^3 x \, dx$

$\int \sin^3 x \cos x \, dx$

Let $u = \sin x \rightarrow \frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$\int \sin^3 x \cos x \, dx$

$= \int u^3 \, du$

Using reverse power rule

$u^3 du = \frac{u^{3+1}}{3+1} + c \Rightarrow \frac{u^4}{4} + c$

hence $u = \sin x$