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30/9/20

MAT 104 Assignment

19/MH301/168

(1) $\int \sin^6 x\, dx$

Using reduction formula:

$$\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\, dx$$

$$\int \sin^6 x\, dx = -\frac{1}{6}\sin^5 x\cos x + \frac{5}{6}\int \sin^4 x\, dx \quad \text{—— (i)}$$

$$\frac{5}{6}\int \sin^4 x\, dx = \frac{5}{6}\left[-\frac{1}{4}\sin^3 x\cos x + \frac{3}{4}\int \sin^2 x\, dx\right]$$

$$\frac{5}{6}\int \sin^4 x\, dx = -\frac{5}{24}\sin^3 x\cos x + \frac{15}{24}\int \sin^2 x\, dx \quad \text{—— (ii)}$$

$$\frac{15}{24}\int \sin^2 x\, dx = \frac{15}{24}\int \frac{1-\cos 2x}{2}$$

$$\frac{15}{24}\int \sin^2 x\, dx = \frac{15}{24}\left[\int \frac{1}{2} - \int \frac{\cos 2x}{2}\right]$$

$$= \frac{15}{24}\left[\frac{x}{2} - \frac{\sin 2x}{4}\right]$$

$$\frac{15}{24}\int \sin^2 x\, dx = \frac{15x}{48} - \frac{15\sin 2x}{96} \quad \text{—— (iii)}$$

Substituting eq (i) & (ix) into eq (i)

$$\int \sin^6 x\,dx = -\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x + \frac{\cancel{15}^{5}}{\cancel{48}_{16}}x - \frac{\cancel{15}^{5}}{\cancel{96}_{32}}\sin 2x$$

$$\therefore \int \sin^6 x\,dx = -\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x + \frac{5x}{16} - \frac{5\sin 2x}{32}$$

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(2) $\int \cos^4 x \sin^3 x\,dx$

let $u = \cos x$ $\quad \frac{du}{dx} = -\sin x$, $dx = -\frac{du}{\sin x}$

$$\int u^4 \sin^3 x\,dx = \int u^4 \sin^3 x \cdot \frac{-du}{\sin x} = \int -u^4 \sin^2 x\,du$$

$$= \int -u^4(1-\cos^2 x)\,du = \int -u^4(1-u^2)\,du$$

$$= \int -u^4 + u^6\,du = \int u^6 - u^4\,du$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\therefore \int \cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

(3) $\int \cos x \sin^3 x \, dx$

let $u = \sin x$ $\quad \frac{du}{dx} = \cos x$, $dx = \frac{du}{\cos x}$

$\int \cos x \, u^3 \, dx = \int \cancel{\cos x} \, u^3 \cdot \frac{du}{\cancel{\cos x}} = \int u^3 \, du$

$= \frac{u^4}{4} + c$

$\therefore \int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$