

Name: Abubakar Zinat

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Integrate the following functions:

① $2x^2 \ln x$

② $3te^{2t}$

③ $x^2 \sin x$

④ $\cos 5x \cos 6x$

⑤ $\sin 7x \cos 2x$

Solution.

① $\int 2x^2 \ln x \, dx$

let $u = \ln x$, $du = \frac{1}{x} dx$, $dv = 2x^2$
 $v = \frac{2x^3}{3}$

$\int u dv = uv - \int v du$

$= \ln x \left(\frac{2x^3}{3} \right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

Then: $\frac{2x^3}{3} \ln x - \frac{2x^2}{3} dx$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$

② $\int 3te^{2t} dt$; let $u = 3t$, $du = 3 dt$, $dv = e^{2t}$, $v = \frac{1}{2} e^{2t}$

$\int u dv = uv - \int v du$

$= 3t \left(\frac{1}{2} e^{2t} \right) - \int \frac{1}{2} e^{2t} \cdot 3 dt$

$= \frac{3t}{2} e^{2t} - \frac{3}{4} e^{2t}$

$= \frac{3t}{2} e^{2t} - \frac{3}{4} e^{2t} + C$

$$\int x \sin x \, dx$$

let $u = x^2$, $dv = \frac{\sin x}{x}$
 $\frac{du}{dx} = 2x$, $v = -\frac{\cos x}{x}$

$$\int u \, dv = uv - \int v \, du$$

$$= x^2 \left(\frac{\cos x}{x} \right) - \int \frac{\cos x}{x} \cdot 2x \, dx$$

$$= x^2 \cos x - \int \cos x \left[\frac{du}{2x} \times \frac{2x}{1} \right]$$

$$\int u \, dv = x^2 \cos x - \int \cos x \, dx = x^2 \cos x - \sin x + c$$

$$\textcircled{4} \int \cos 5x \cos 6x \, dx$$

let $A = 5x$, $B = 6x$
 $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

$$\cos A \cos B = \frac{1}{2} [\cos(11x) + \cos(-x)]$$

$$= \frac{1}{2} \left[\frac{\cos 11x}{11} + \frac{\cos(-x)}{1} \right]$$

$$\int \cos 5x \cos 6x \, dx = \frac{\cos 11x}{22} + \frac{\cos(-x)}{2} + C$$

$$\textcircled{5} \int \sin 7x \cos 2x \, dx$$

let $A = 7x$ and $B = 2x$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$= \frac{1}{2} \left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right]$$

$$\int \sin 7x \cos 2x \, dx = \frac{\sin 9x}{18} + \frac{\sin 5x}{10} + c$$