

19 / MATHS 01 / 061

MBBS

1) $\cos x \sin^3 x$

$$u = \sin x \Rightarrow du \Rightarrow \cos x dx$$

$$\int \cos x \sin^3 x dx = \int u^3 du = \frac{1}{4} u^4 + C$$
$$= \frac{1}{4} \sin^4 x + C$$

2) $\cos^4 x \sin^3 x = \int \sin^2 x \cos^4 x \sin x dx$

$$\Rightarrow \sin x \Rightarrow du \Rightarrow \cos x dx$$

$$\int \cos^4 x \sin^3 x = \int u^3$$

$$\text{Let } u = \cos x \Rightarrow du \Rightarrow -\sin x dx \Rightarrow$$
$$= \sin x dx$$

$$\text{Also, } \sin^2 x = 1 - \cos^2 x$$

$$= 1 - u^2$$

$$\text{as } \cos^4 x = u^4$$

substituting, we have

$$= \int (1 - u^2) u^4 du = \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$$

3)

a) $\sin^6 x$

Using Reduction formula

$$u = \sin x \Rightarrow du = \cos x dx$$

$$S_n = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} S_{n-2}$$

$$S_6 = \frac{-\sin^{6-1} x \cos x}{6} + \frac{6-1}{6} S_4$$

19/MAS01/061

MBBS

1) $\int \cos x \sin^3 x \, dx$

$$u = \sin x \Rightarrow du \Rightarrow \cos x \, dx$$

$$\int \cos x \sin^3 x \, dx = \int u^3 \, du = \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} \sin^4 x + C$$

2) $\int \cos^4 x \sin^2 x \, dx = \int \sin^2 x \cos^4 x \sin x \, dx$

$$\rightarrow \sin x \Rightarrow du \Rightarrow \cos x \, dx$$

$$\int \cos^4 x \sin^2 x \, dx = \int u^2 \, du$$

$$\text{Let } u = \cos x \Rightarrow du \Rightarrow -\sin x \, dx \Rightarrow -du$$

$$= \sin x \, dx$$

Also, $\sin^2 x = 1 - \cos^2 x$

$$= 1 - u^2$$

as $\cos^4 x = u^4$

substituting, we have

$$= \int (1 - u^2) u^4 \, du = \int (u^6 - u^4) \, du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$$

$\int \sin^6 x \, dx$

Using Reduction formula

$$u = \sin x \Rightarrow du = \cos x \, dx$$

$$I_n = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

$$I_6 = \frac{-\sin^{5} x \cos x}{6} + \frac{5}{6} I_4$$

$$I_4 = \frac{-\sin^3 x \cos x}{4} + \frac{3}{4} I_2$$

Similarly, we obtain

expression for I_2

$$\cos^4 x \sin^3 x = \int \sin^2 x \cos^4 x \sin x dx$$

$$\Rightarrow \sin x \Rightarrow du \Rightarrow \cos x dx$$

$$\int \cos^4 x \sin^3 x = \int u^3$$

$$\text{Let } u = \cos x \Rightarrow du \Rightarrow -\sin x dx \Rightarrow -du$$

$$= \sin x dx$$

$$\text{Also, } \sin^2 x = 1 - \cos^2 x$$

$$= 1 - u^2$$

$$\text{as } \cos^4 x = u^4$$

substituting, we have

$$\int (1 - u^2) u^4 du = \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5}$$

$$= \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$$

3)

a) $\int \sin^6 x$

Using Reduction formula

$$u = \sin x \Rightarrow du = \cos x dx$$

$$S_n = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} S_{n-2}$$

$$S_6 = \frac{-\sin^{6-1} x \cos x}{6} + \frac{6-1}{6} S_{6-2}$$

$$S_6 = \frac{-\sin^5 x \cos x}{6} + \frac{5}{6} S_4$$

Similarly, we obtain expression for S_4

$$S_4 = \frac{-\sin^3 x \cos x}{4} + \frac{3}{4} S_2$$

Substitute the above S_4 in S_6

$$S_4 = \frac{\sin^3 x \cos x}{6} = \frac{5}{6}$$

$$\left(\frac{\sin^3 x \cos x}{4} - \frac{3 \sin x \cos x}{4} + \frac{3/4}{8} \right)$$

$$S_6 = \frac{\sin^5 x \cos x}{6} - \frac{5 \sin^3 x \cos x}{24} - \frac{\sin x \cos x}{16} + \frac{3/4}{8}$$