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MAT 104

1. $\int \sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1-2\cos 2x+\cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2-4\cos 2x+1+\cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3-4\cos 2x+\cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 2x-\cos 4x\cos 2x$

$= \frac{1}{16}(3-7\cos 2x-\cos 4x+2(2\cos^2 2x)-\frac{1}{2}2\cos 4x\cos 2x$

$= \frac{1}{16}\left[3-7\cos 2x+\cos 4x+2(1+\cos 4x)-\frac{1}{2}(\cos 6x+\cos 2x)\right]$

$= \frac{1}{16}\left[3-7\cos 2x+\cos 4x+2+2\cos 4x-\frac{1}{2}(\cos 6x+\cos 2x)\right]$

$= \frac{1}{32}\left[6-14\cos 2x+2\cos 4x+4+4\cos 4x-\cos 6x-\cos 2x\right]$

$= \frac{1}{32}\left[10-15\cos 2x+6\cos 4x-\cos 6x\right]$

let $\sin^6 x = R$

$R = \frac{1}{32}\int (10-15\cos 2x+6\cos 4x-\cos 6x)\,dx$

$R = \frac{1}{32}\left(10x-\frac{15\sin 2x}{2}+\frac{6\cos 4x}{4}-\frac{\cos 6x}{6}\right)+C$

$\int \sin^6 x = \frac{10x}{32}-\frac{15\sin 2x}{64}+\frac{6\cos 4x}{128}-\frac{\cos 6x}{192}+C$

② $\cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x = -\int \cancel{\sin x} \cdot \sin^2 x \, u^4 \frac{-du}{\cancel{\sin x}}$

$= -\int (1-u^2) u^4 \, du$

$= -\int u^4 - u^6 \, du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7} \right] + C$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

$\int \cos^4 x \sin^3 x = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

③ $\cos x \sin^3 x$

$u = \sin x$

$du = \cos x \, dx$

$\int \cos x \sin^3 x \, dx = \int u^3 \, du$

$= \frac{1}{4} \sin^4 x + C$