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Math 104

Assignment

① $\sin^6 x$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$\left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$\frac{1}{8}\left(1-2\cos 2x+\cos^2 2x\right)\left(1-\cos 2x\right)$$

$$\frac{1}{8}\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)\left(1-\cos 2x\right)$$

$$\frac{1}{16}\left(2-4\cos 2x+1+\cos 4x\right)\left(1-\cos 2x\right)$$

$$\frac{1}{16}\left(3-4\cos 2x+\cos 4x\right)\left(1-\cos 2x\right)$$

$$\frac{1}{16}\left(3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 2x-\cos 4x\cos 2x\right)$$

$$\frac{1}{16}\left(3-7\cos 2x+\cos 4x+(2(2\cos^2 2x)-\tfrac{1}{2}\,2\cos 4x\cos 2x\right)$$

$$\frac{1}{16}\left[3-7\cos 2x+\cos 4x+2(1+\cos 4x)-\tfrac{1}{2}(\cos 6x+\cos 2x)\right]$$

$$\frac{1}{16}\left[3-7\cos 2x+\cos 4x+2+2\cos 4x-\frac{1}{2}(\cos 6x+\cos 2x)\right]$$

$$\frac{1}{32}\left[6-14\cos 2x+2\cos 4x+4+4\cos 4x-\cos 6x-\cos 2x\right]$$

$$\frac{1}{32}\left[10-15\cos 2x+6\cos 4x-\cos 6x\right]$$

Let $\sin^6 x = R$

$$R=\frac{1}{32}\int(10-15\cos 2x+6\cos 4x-\cos 6x)\,dx$$

$$R=\frac{1}{32}\left(10x-\frac{15\sin 2x}{2}+\frac{6\cos 4x}{4}-\frac{\cos 6x}{6}\right)+C$$

Mathematics

$\cos^4 x \sin^3 x$

$u = \cos x$

$du/dx = -\sin x \implies dx = -du/\sin x$

And

$\sin^2 x = 1 - \cos^2 x$

$\int \cancel{\sin x} \cdot \sin^2 x \cdot u^4 \cdot \dfrac{-du}{\cancel{\sin x}}$

$= -\int (1 - \cos^2 x) \cdot u^4 \cdot du$

$-\int u^4 (1 - u^2) \cdot du$

$= -\int (u^4 - u^6) \cdot du$

$= -\left[\dfrac{u^5}{5} - \dfrac{u^7}{7}\right] du + C$

$\dfrac{u^7}{7} - \dfrac{u^5}{5} + C$

$\dfrac{(\cos x)^7}{7} - \dfrac{(\cos x)^5}{5} + C$

(3) $\cos x \sin^3 x$

$u = \sin x$

$du/dx = \cos x \therefore dx = du/\cos x$

$\int \cos x \cdot u^3 \cdot \dfrac{du}{\cos x}$

$= \int u^3 \cdot du = \dfrac{u^4}{4} + C$

$= \dfrac{\sin^4 x}{4} + C$

$$\sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + c$$