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Integrate the following functions.

1) $\int \sin^6 x \, dx$

$\int \sin^2 x \cdot \sin^4 x \, dx$

$$\int \left[\frac{1-\cos 2x}{2}\right]\left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)dx$$

$$\frac{1}{8}\int\left[1(1-2\cos 2x+\cos^2 2x)-\cos 2x(1-2\cos 2x+\cos^2 2x\right]dx$$

$$\frac{1}{8}\int\left[1-2\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x\right]dx$$

$$\frac{1}{8}\int\left[1-2\cos 2x-\cos 2x+\cos^2 2x+2\cos^2 2x-\cos^3 2x\right]dx$$

$$\frac{1}{8}\int\left[1-3\cos 2x+3\cos^2 2x-\cos^3 2x\right]dx$$

$$\frac{1}{8}\int\left[1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right)-\cos 2x+\cos 2x\sin^2 2x\,dx\right.$$

$$\frac{1}{8}\int 3\left(\frac{1}{2}+\frac{\cos 4x}{2}\right) - 4\cos 2x + \cos 2x \sin^2 2x \, dx$$

$$= \frac{1}{8}\left[3\left(\frac{x}{2}+\frac{\sin 4x}{8}\right) + \frac{\sin^3 2x}{6}\right] + C$$

2) $\int \cos^4 x \sin^3 x \, dx$

Solution: $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\int u^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$$

$$-\int \sin^2 x \cdot u^4 \, du$$

$$-\int (1 - \cos^2 x) u^4 \, du$$

$$\int (-1 + \cos^2 x) u^4 \, du$$

$$\int (\cos^2 x - 1) u^4 \, du$$

$$\int (u^2 - 1) u^4 \, du$$

$$\int (u^6 - u^4) \, du$$

$$= \left[\frac{u^7}{7} - \frac{u^4}{4}\right] + C$$

$$= \left[\frac{(\cos x)^7}{7} - \frac{(\cos x)^4}{4}\right] + C$$

3) $\int \cos x \sin^3 x \, dx$.

Solution: $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$= \int u^3 \, du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{(\sin x)^4}{4} + C$$

$$= \frac{(\sin x)^4}{4} + C$$