$\int \cos^4 x \sin^3 x \, dx$

Soln

since m is odd, $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -du/\sin x$

$\int \cos^4 x \sin^3 x \, dx = \int \cos^4 x \sin x \cdot \sin^2 x \, dx$

$= \int \cos^4 x \sin x (1 - \cos^2 x) dx$

$= \int u^4 (1 - x^2) \, -du/\sin x$

$= -\int u^4 (1 - u^2) du$

$= -\int (u^4 - u^6) du$

$= -\left[u^5/5 - u^7/7 \right] + c$

$- u^5/5 + u^7/7 + c$

$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

$\int \cos x \sin^3 x \, dx$

Soln

m is odd, $u = \cos x$

$du/dx = -\sin x$

$dx = -du/\sin x$

$\int \cos x \sin^3 x \, dx = \int \cos - (1 - \cos^2 x) - du/\sin x$

$= -\int u(1 - u^2) du$

$= -\int (u - u^3) du = \int (u^3 - u) du$

$= u^4/4 - u^2/2 + c$

$\therefore \int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$

$\int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$

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1. $\int \sin^6 x \, dx$

Solution

$\int \sin^6 x \, dx = \int (\sin^2 x)^3 \, dx$

$= \int \left(\frac{1-2\cos 2x}{2}\right)^3 dx$

$= \frac{1}{8}\int (1-2\cos 2x)^3 = \frac{1}{8}\int (1-2\cos 2x)^2 (1-2\cos 2x) \, dx$

$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x)(1-2\cos 2x) \, dx$

$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x - 2\cos 2x + 8\cos^2 2x - 8\cos^3 2x)$

$= \frac{1}{8}\int (1 - 6\cos 2x + 12\cos^2 2x - 8\cos^3 2x) \, dx$

$= \frac{1}{8}\int (1 - 6\cos 2x + 12\left(\frac{1+\cos 4x}{2}\right) - 8\cos^3 2x) dx$

$= \frac{1}{8}\left[\int dx - 6\int \cos 2x \, dx + 6\int (1+\cos 4x) dx - 8\int \cos^3 2x \, dx\right]$

but $\int \cos^3 2x \, dx = \int \cos^2 2x \cos 2x \, dx$

$= \int \frac{\cos 2x (1+\cos 4x)}{2} dx$

$= \frac{1}{2}\int [\cos 2x + \cos 2x \cos 4x] \, dx$

from $\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$

$\cos 2x \cos 4x = \frac{1}{2}(\cos 6x + \cos 2x)$

$= \int \cos^3 2x \, dx = \frac{1}{2}\left[\int \cos 2x + \frac{1}{4}(\cos 6x + \cos 2x) dx\right]$

$= \frac{1}{2}\left[\cos 2x \, dx + \frac{1}{4}\int \cos 6x \, dx + \frac{1}{4}\int \cos 2x \, dx\right.$

$= \frac{1}{2} \cdot \frac{\sin 2x}{2} + \frac{1}{4} \frac{\sin 6x}{6} + \frac{1}{4} \cdot \frac{\sin 2x}{2}$

$= \frac{1}{2} \cdot \frac{\sin 2x}{2} + \frac{1}{4} \frac{\sin 6x}{6} + \frac{1}{4} \cdot \frac{\sin 2x}{2}$

$\int \cos^3 2x \, dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}$

$\int \sin^6 x \, dx = \frac{1}{8}\left[\int dx - 6\int \cos 2x \, dx + 6\int (1+\cos 4x) dx - 8\int \cos^3 2x \, dx\right]$

$= \frac{1}{8}\left(x - \frac{{}^3\cancel{6}\sin 2x}{\cancel{2}} + 6x + \frac{{}^3\cancel{6}\sin 4x}{\cancel{4}_2} - 8\left(\frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}\right)\right]$

$= \frac{1}{8}x - \frac{3}{8}\sin 2x + \frac{3}{4}x + \frac{3}{16}\sin 4x - \sin 2x/4 - \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + C$

$= 7x/8 - \frac{6\sin 2x}{8} + \frac{13\sin 4x}{16} - \frac{\sin 6x}{24} + C$

$\therefore$ ~~$\int \sin^6 x \, dx = 7x/8 -$~~

$\therefore \int \sin^6 x \, dx = \frac{7x}{8} - \frac{6\sin 2x}{8} + \frac{13}{16}\sin 4x - \frac{\sin 6x}{24} + C$