MAT 104

ASSIGNMENT

1. $\sin^6 x$

$\sin^2 x = 1-\cos^2 x$

$\int (1-\cos^2 x)^3\, dx$

$\int (1-\cos^2 x)^2 (1-\cos^2 x)\, dx$

$\int (1-2\cos^2 x + \cos^4 x)(1-\cos^2 x)\, dx$

$\int 1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x\, dx$

Using $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\int 1 - 3\left(\frac{1}{2}(1+\cos 2x)\right) + 3\left(\frac{1}{4}(1+\cos 2x)^2\right) - \left(\frac{1}{8}(1+\cos 2x)^3\right) dx$$

$$\int 1 - \frac{3}{2}(1+\cos 2x) + \frac{3}{4}(1+\cos 2x)^2 - \frac{1}{8}(1+\cos 2x)^3\, dx$$

$$\int 1 - \frac{3}{2} - \frac{3}{2}\cos 2x + \frac{3}{4}(1+2\cos 2x+\cos^2 2x) - \frac{1}{8}(1+3\cos 2x+3\cos^2 2x+\cos^3 2x)\, dx$$

$$\int 1 - \frac{3}{2} - \frac{3}{2}\cos 2x + \frac{3}{4} + \frac{3}{2}\cos 2x + \frac{3}{4}\cos^2 2x - \frac{1}{8} - \frac{3}{8}\cos 2x - \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x\, dx$$

$$\int -\frac{1}{2} - \frac{3}{2}\cos 2x + \frac{3}{4} + \frac{3}{2}\cos 2x + \frac{3}{4}\cos^2 2x - \frac{1}{8} - \frac{3}{8}\cos 2x - \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x\, dx$$

$$\int \frac{1}{8} + \frac{3}{4}\cos^2 2x - \frac{3}{8}\cos 2x - \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x\, dx$$

$$\int \frac{1}{8} + \frac{3}{4}\left(\frac{1+\cos 4x}{2}\right) - \frac{3}{8}\cos 2x - \frac{3}{8}\left(\frac{1+\cos 4x}{2}\right) - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{1}{8} + \frac{3}{8} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16} - \frac{3}{16}\cos 4x - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16}\cos 4x - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16}\cos 4x - \frac{1}{8}(1-\sin^2 2x)(\cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16}\cos 4x - \frac{1}{8}\int (1-\sin^2 2x)(\cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16}\cos 4x$$

$= \frac{5x}{16} + \frac{3}{32}\sin 4x - \frac{3}{16}\sin 2x - \frac{3}{64}\sin 4x$

Finding $\int (1-\sin^2 2x)(\cos 2x)$

let $y = \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

$\frac{1}{2} dy = \cos 2x \, dx$

$\frac{1}{2}\int 1 - y^2 \, dy$

$= \frac{1}{2}\left(y - \frac{y^3}{3}\right)$

$= \frac{y}{2} - \frac{y^3}{6}$

$= \frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}$

Add everything

$= \frac{5x}{16} + \frac{3}{32}\sin 4x - \frac{3}{16}\sin 2x - \frac{3}{64}\sin 4x - \frac{1}{8}\left(\frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}\right) + C$

$= \frac{5x}{16} + \frac{3}{32}\sin 4x - \frac{3}{16}\sin 2x - \frac{3}{64}\sin 4x - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48} + C$

$\int \sin^6 x = \frac{60x + 18\sin 4x - 36\sin 2x - 9\sin 4x - 12\sin 2x + 4\sin^3 2x}{192} + C$

$\int \sin^6 x = \frac{60x + 9\sin 4x - 48\sin 2x + 4\sin^3 2x}{192} + C$ 15x 60x

2. $\int \cos^4 x \sin^3 x \, dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$ $dx = -\frac{dy}{\sin x}$

$\int u^4 \cdot \sin^3 x \cdot -\frac{du}{\sin x}$

$-\int u^4 \cdot \sin^2 x \, du$

$\sin^2 x = 1 - \cos^2 x$

$-\int u^4(1 - u^2) \, du$

$-\int u^4 - u^6 \, du$

$= \int u^6 - u^4 \, du$