(3) $\int \cos x \sin^3 x =$ let $u = \cos x$

$$\frac{du}{dx} = -\sin x \qquad dx = \frac{du}{-\sin x}$$

$$\int \cos x \sin^3 x = \int u \sin^3 x \frac{du}{-\sin x}$$

$$\int \cos x \sin^3 x = -\int u \sin^2 x \, du$$

$$= -\int u(1-\cos^2 x)\, du$$

$$\int \cos x \sin^3 x = -\int u(1-u^2)\, du$$

$$\int \cos x \sin^3 x = -\int u - u^3 \, du$$

$$-\left[\frac{u^2}{2} - \frac{u^4}{4}\right]$$

$$\int \cos x \sin^3 x = \left[\frac{u^4}{4} - \frac{u^2}{2}\right] + c$$

$$\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$$

$$\int \sin^4 x = \frac{1}{8}\left(\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{8} - \left(\frac{u}{2} - \frac{u^3}{6}\right)\right) + c$$

since $u = \sin 2x$

$$\int \sin^4 x = \frac{1}{8}\left(\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{8} - \frac{\sin 2x}{2} + \frac{\sin^3 2x}{6}\right) + c$$

$$\cancel{\frac{1}{8}\;\frac{5x}{16} - \frac{3\sin 2x}{16} + \frac{3\sin 4x}{64}} -$$

$$\int \sin^4 x = \frac{1}{16}\left(5x - 3\sin 2x + \frac{3\sin 4x}{4} - \sin 2x + \frac{\sin^3 2x}{3}\right) + c$$

$$\int \sin^4 x = \frac{1}{16}\left(5x - 4\sin 2x + \frac{3\sin 4x}{4} + \frac{\sin^3 2x}{3}\right) + c$$

(2) $\int \cos^4 x \sin^3 x =$ let $u = \cos x$ $\quad \frac{du}{dx} = -\sin x \quad dx = \frac{du}{-\sin x}$

$$\int \cos^4 x \sin^3 x = \int u^4 \sin^3 x \frac{du}{-\sin x}$$

$$\int \cos^4 x \sin^3 x = -\int u^4 \sin^2 x \, du$$

$$\int \cos^4 x \sin^3 x = -\int u^4 (1 - \cos^2 x)\, du = -\int u^4(1-u^2)\, du$$

$$\int \cos^4 x \sin^3 x = -\int u^4 - u^6 \, du = \int u^6 - u^4 \, du$$

$$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + c$$

$$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

$$\int \sin^6 x = \int (\sin^2 x)^3$$

$$\int \sin^6 x = \int (\sin^2 x)^2 \sin^2 x$$

$$= \int \left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right)\left(\frac{1 - \cos 2x}{2}\right)$$

$$=$$

$$\int \sin^6 x = \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$$

$$\int \sin^6 x = \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x + \cos^3 2x)$$

$$\int \sin^6 x = \frac{1}{8}\int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)$$

$$\int \sin^6 x = \frac{1}{8}\int \left(1 - 3\cos 2x + 3\left(\frac{1 + \cos 4x}{2}\right) - (\cos^2 2x)\cos 2x\right)$$

$$\int \sin^6 x = \frac{1}{8}\left(\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \int\left[(1 - \sin^2 2x)\cos 2x\right]\right)$$

$$\int \sin^6 x = \frac{1}{8}\left(\int \frac{5}{2} - 3\cos 2x + \frac{3\cos 4x}{2} - \int (1 - \sin^2 2x)\cos 2x\right)$$

let $u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$

$dx = du / 2\cos 2x$

$$\int \sin^6 x = \frac{1}{8}\left(\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3}{8}\sin 4x - \int (1 - u^2)\cos 2x \frac{du}{2\cos 2x}\right)$$

$$\int \sin^6 x \quad \frac{1}{8}\left(\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{8} - \int \frac{1}{2} - \frac{u^2}{2}\, du\right.$$