

Name: OLUOTHO BASHIRAT BOLAJOKO

Matric number: 19/MTH01/371

Department: Medicine and surgery

Course: MAT 101

Assigned

Integrate the following functions:

1.) $\sin^6 x$:

$$\int \sin^6 x dx$$

$$\sin^6 x = \sin^2 x \cdot \sin^4 x dx$$

$$\int \sin^6 x = \int \frac{1 - \cos 2x}{2} \cdot \frac{1 - 2\cos 2x + \cos^2 2x}{4} dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x - \cos^3 2x dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3 \left(\frac{1 + \cos 4x}{2} \right) - \cos 2x (1 - \sin^2 x) dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 x dx$$

$$\int \sin^6 x = \frac{1}{8} \int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x dx$$

$$\int \sin^6 x = \frac{1}{8} \int \frac{5}{2} x - \frac{4\sin 2x}{2} + \frac{3\sin 4x}{2} \times \frac{1}{4} + \int \cos 2x \sin^2 2x dx$$

(ABUAD), The Road to Intellectualism, Quality and Excellence

$$\int \cos 2x \sin^2 2x dx = \int \cos 2x (u)^2 \frac{du}{2 \cos 2x}$$

$$\begin{aligned} \text{Let } u &= \sin 2x \\ \frac{du}{dx} &= 2 \cos 2x \\ \therefore dx &= \frac{du}{2 \cos 2x} \end{aligned}$$

$$\int \cos 2x \sin^2 2x dx = \int u^2 \frac{du}{2}$$

$$\int \cos 2x \sin^2 2x dx = \frac{1}{2} \int u^2 du$$

$$\int \cos 2x \sin^2 2x dx = \frac{1}{2} \left(\frac{u^3}{3} + C \right)$$

$$\int \cos 2x \sin^2 2x dx = \frac{1}{2} \left(\frac{\sin^3 2x}{3} + C \right)$$

$$\therefore \int \sin^6 x dx = \frac{1}{8} \left[\frac{5x}{2} - 2 \sin 2x + \frac{3 \sin 4x}{8} + \frac{\sin^3 2x}{6} \right] + C$$

$$2.) \cos^4 x \sin^3 x$$

$$\cos^4 x \sin^3 x = \int \cos^4 x \sin^2 x \sin x dx$$

$$\sin^2 x + \cos^2 x = 1$$

$$\therefore \sin^2 x = 1 - \cos^2 x$$

$$\cos^4 x \sin^3 x = \int \cos^4 x (1 - \cos^2 x) \sin x dx$$

$$\text{Let } u = \cos^2 x$$

$$du = -2 \cos x \sin x dx$$

$$\cos^4 x \sin^3 x = -\int u^2 (1 - u) du$$

$$\cos^4 x \sin^3 x = -\int u^2 - u^3 du$$

$$\cos^4 x \sin^3 x = -\frac{u^3}{3} + \frac{u^4}{4} + C$$

$$\cos^4 x \sin^3 x = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

$$3.) \cos x \sin^3 x =$$

$$\text{Let } u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$\int \cos x \sin^3 x dx = \int \cancel{\cos x} u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$\therefore \int \cos x \sin^3 x dx = \int u^3 du$$

$$\int \cos x \sin^3 x dx = \frac{u^4}{4} + C$$

$$\therefore \int \cos x \sin^3 x dx = \frac{\sin^4 x}{4} + C$$