ADELEKE BLESSING OLUWAKEMI

MHS/MBBS

19/MHS01/030

1) $\sin^6 x \, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8} (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8} \left[1 - 2\cos 2x + \left[\frac{1+\cos 4x}{2}\right]\right] [1 - \cos 2x]$

$= \frac{1}{16} [2 - 4\cos 2x + 1 + \cos 4x] [1 - \cos 2x]$

$= \frac{1}{16} [3 - 4\cos 2x + \cos 4x] [1 - \cos 2x]$

$= \frac{1}{16} [3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x$

$- \cos 4x \times \cos 2x$

$= \frac{1}{16} [3 - 7\cos 2x + \cos 4x + 2 [2\cos^2 2x]$

$- \frac{1}{2} 2\cos 4x \times \cos 2x$

$= \frac{1}{16} [3 - 7\cos 2x + \cos 4x + 2 [1 + \cos 4x]$

$- \frac{1}{2} [\cos 6x + \cos 2x]$

$= \frac{1}{16} [3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}$

$[\cos 6x + \cos 2x]]$

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$$= \frac{1}{32}[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$$

$$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$$

$\sin^6 x$

$$= \frac{1}{32}\int [10x - 15\cos 2x + 6\cos 4x - \cos 6x]\,dx$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right] + c$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + c$$

2] $\cos^4 x \sin^3 x \quad - \cos^n x \sin^m x \, dx$

$\int \cos^4 x \sin^3 x \, dx$

since m is odd

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$

And

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$= \int \cos^4 x \sin x \cdot \sin^2 x$

$= \int u^4 \sin x \cdot 1 - \cos^2 x \cdot \frac{-du}{\sin x}$

$= -\int u^4 (1 - \cos^2 x) \, du$

$= -\int (1 - \cos^2 x) u^4 \, du$

$= \int (\cos^2 x - 1) u^4 \, du$

$= \int (u^2 - 1) u^4 \, du$

$= \int (u^6 - u^4) \, du$

$= \left[\frac{u^7}{7} - \frac{u^5}{5} \right] + c$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

3] $\cos x \sin^3 x$ — $\cos^n x \sin^m x$

Since m is odd

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$

And

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$= \int \cos x \cdot \sin x \cdot \sin^2 x \, dx$

$= \int u \cdot \sin x \cdot [1 - \cos^2 x] \frac{-du}{\sin x}$

$= \int u [1 - \cos^2 x] - du$

$= \int u [1 - u^2] - du$

$= - \int [u - u^3] - du$

$= \int [u^3 - u] \, du$

$= \left[\frac{u^4}{4} - \frac{u^2}{2}\right] + c$

$= \left[\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2}\right] + c$