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MEDICINE AND SURGERY

MATHS 104

MATRIC No: 19/mhs01/418

Integrate the following functions:

1 $\sin^6 x$

$$\int \sin^6 x$$

$$= \int (\sin^2 x)^2 (\sin^2 x)$$

$$= \int \frac{1}{4}(1-\cos 2x)^2 \cdot \frac{1}{2}(1-\cos 2x)$$

$$= \int \frac{1}{4}(1-2\cos 2x+\cos^2 2x) \cdot \frac{1}{2}(1-\cos 2x)$$

$$= \int \frac{1}{8}(1-2\cos 2x+\cos^2 2x)(1-\cos 2x)$$

$$= \int \frac{1}{8}\left(1-2\cos 2x+\frac{\cos 4x}{2}+\frac{1}{2}\right)(1-\cos 2x)$$

$$= \int \frac{1}{8}\left(\frac{2-4\cos 2x+\cos 4x+1}{2}\right)(1-\cos 2x)$$

$$= \frac{1}{16}\int (2-4\cos 2x+\cos 4x+1)(1-\cos 2x)$$

$$= \frac{1}{16}\int (3+\cos 4x-4\cos 2x-3\cos 2x+4\cos^2 2x-\cos 2x\cos 4x)$$

$$= \frac{1}{16}\int (3-7\cos 2x+\cos 4x+2(1+\cos 4x)-\frac{1}{2}(\cos 6x+\cos 2x))$$

$$= \frac{1}{32}\int 6-14\cos 2x+2\cos 4x+4+4\cos 4x(-\cos 6x-\cos 2x)$$

$$= \frac{1}{32}\int (10-15\cos 2x+6\cos 4x-\cos 6x)$$

3 $\int \cos x \sin^3 x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$du = \cos x \, dx$

$= \int u^3 \cdot du$

$= \frac{u^4}{4} \cdot du$

$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6} + c\right]$$

$$\int \sin^6 x = \frac{1}{192}\left(10x - 15\sin 2x + 9\sin 4x - \sin 6x\right) + C$$

2 $\int \cos^4 x \sin^3 x \, dx$

Let $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = 1 - u^2$$

$$\int \cos^4 x \sin^3 x \, dx = \int \sin^2 x \cdot u^4 \cdot \sin x \cdot \frac{-du}{\sin x}$$

$$= \int -\sin^2 x \cdot u^4 \cdot du$$

$$= -\int (1 - u^2) \cdot u^4 \cdot du$$

$$= -\int u^4 - u^6 \cdot du$$

$$= -\left[\frac{u^5}{5} - \frac{u^7}{7} \cdot du\right]$$

$$= \frac{u^7}{7} - \frac{u^5}{5} \cdot du$$

$$\int \cos^4 x \sin^3 x \, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$