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Course: Math 104

Assignment

1) $\int \sin^6 x\,dx$

$\int \sin^6 x\,dx = \int \sin^2 x \cdot (\sin^2 x)^2\,dx$

Since $\sin^2 x = \frac{1-\cos 2x}{2}$

$\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right) dx$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)^2 \cdot dx$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right) \cdot dx$

$= \frac{1}{8}\int (1-\cos 2x)(1-2\cos 2x+\cos^2 2x) \cdot dx$

$= \frac{1}{8}\int (1-2\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x) \cdot dx$

$= \frac{1}{8}\int (1-3\cos 2x+3\cos^2 2x-\cos^3 2x)\,dx$

$= \frac{1}{8}\int \left[1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right)-\cos 2x\,(1+\sin^2 2x)\right] dx$

$= \frac{1}{8}\int \left[1-3\cos 2x+\frac{3}{2}+\frac{3\cos 4x}{2}-\cos 2x+\cos 2x \sin^2 2x\right] dx$

$= \frac{1}{8}\int \frac{5}{2}-2\cos 2x-4\cos 2x+\frac{3\cos 4x}{2}+\cos 2x \sin^2 2x\,dx$

$\int \sin^6 x\,dx = \frac{1}{8}\left[\frac{5}{2}x-2\sin 2x-\frac{3\sin 4x}{8}+\frac{3\sin^3 2x}{6}\right]+C$

2) $\int \cos^4 x \sin^3 x$

$u = \cos x$

$du/dx = -\sin x \;;\; dx = -\frac{du}{\sin x}$

Since $\sin^2 x = 1-\cos^2 x$

$\int \sin x \cdot \sin^2 x \cdot u^4 \cdot -\frac{du}{\sin x}$

$$\int u^4 \, du$$

$$\int (1-\cos^2 x)^2 \, u^4 \, du$$

$$\int (1+\cos^4 x - 2\cos^2 x) \, u^4 \, du$$

$$\int (u^4 - 1) \, u^4 \, du$$

$$\int (u^8 - u^4) \, du + c$$

$$\frac{u^9}{9} - \frac{u^5}{5} + c$$

$$\frac{\cos^9 x}{9} - \frac{\cos^5 x}{5} + c$$

3) $\cos x \sin^3 x \; du/dx = -\sin x$

$u = \cos x$

$dx = -\frac{du}{\sin x}$

$$\int u \cdot \sin x \cdot (\sin^3 x) \cdot \frac{-du}{\sin x}$$

$$-\int u \cdot (\sin^2 x) \, du$$

$$-\int u \cdot (1-\cos^2 x) \, du$$

$$\int -u \, (1-\cos^2 x) \, du$$

$$\int -u \, (1-u^2) \, du$$

$$\int -u + u^3 \, du$$

$$-\frac{u^2}{2} + \frac{u^4}{4} + c$$

$$\frac{u^4}{4} - \frac{u^2}{2} + c$$

$$\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$$