AKINLADE OLUWANIYI ELISHA 19/MHS01/072 MBBS

1. Find $\int \sin^6 x \, dx$

Recall; $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x + \sin^2 x = 1$

Soln.

$= \int (\sin^2 x)^2 \cdot \sin^2 x \, dx \quad = \int \frac{(1-\cos 2x)^2}{4} \cdot \sin^2 x \, dx$

$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)(0.5(1-\cos 2x))\, dx$

$= \frac{1}{8}\int \left[1 - 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right]\left[1 - \cos 2x\right] dx \quad \left(\cos^2 2x = \frac{1}{2}(1+\cos 4x)\right)$

$= \frac{1}{8}\int \left(\frac{3}{2} - 2\cos 2x + \frac{\cos 4x}{2}\right)\left(1 - \cos 2x\right) dx$

$= \frac{1}{8}\int \left(\frac{3}{2} - \frac{3}{2}\cos 2x - 2\cos 2x + 2\cos^2 2x + \frac{\cos 4x}{2} - \frac{\cos 4x \cos 2x}{2}\right) dx$

Simplifying what's inside the bracket;

$= \frac{3}{2} - \frac{7}{2}\cos 2x + 2 \cdot \frac{1}{2}(1+\cos 4x) + \frac{\cos 4x}{2} - \frac{\cos 4x \cos 2x}{2}$

$= \frac{5}{2} - \frac{7}{2}\cos 2x + \frac{3}{2}\cos 4x - \frac{\cos 4x \cos 2x}{2}$

$\therefore \Rightarrow \frac{1}{8}\left[\int \frac{5}{2} dx - \frac{7}{2}\int \cos 2x \, dx + \frac{3}{2}\int \cos 4x \, dx - \frac{1}{2}\int \cos 4x \cos 2x\right.$

$= \frac{1}{8}\left[\frac{5x}{2} - \frac{7}{4}\sin 2x + \frac{3}{8}\sin 4x - \frac{1}{4}\int (\cos 6x + \cos 2x)\, dx\right]$

$= \frac{1}{8}\left[\frac{5x}{2} - \frac{7}{4}\sin 2x + \frac{3}{8}\sin 4x - \frac{\sin 6x}{24} - \frac{\sin 2x}{8}\right] + C$

$$= \frac{1}{8}\left[\frac{5x}{2} - \frac{15}{8}\sin 2x + \frac{3}{8}\sin 4x - \frac{\sin 6x}{24}\right] + C_n$$

$$= \frac{1}{32}\left[10x - \frac{15}{2}\sin 2x + \frac{3}{2}\sin 4x - \frac{\sin 6x}{6}\right] + C_n$$

2. Find $\int \sin^3 x \cos^4 x \, dx$

Soln.

Let $u = \cos x$

$dx = \frac{-du}{\sin x}$

$$= \int \sin^3 x \cdot u^4 \cdot \frac{-du}{\sin x}$$

$$= \int \sin^2 x \cdot -u^4 \, du$$

$$= \int (1 - \cos^2 x) \cdot -u^4 \, du \quad [\cos^2 x + \sin^2 x = 1]$$

$$= \int (-u^4 + u^6) \, du$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\int \sin^3 x \cos^4 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C \quad [u = \cos x]$$

3. Find $\int \sin^3 x \cos x \, dx$

Soln.

Let $u = \sin x$, $dx = \frac{du}{\cos x}$ $\Rightarrow \int u^3 \cdot \cos x \cdot \frac{du}{\cos x}$

$$\Rightarrow \int u^3 \, du = \frac{u^4}{4} + C$$

$$\int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4} + C_n \quad [u = \sin x]$$