ADEBIYI ADEOLUWA EMMANUEL

19/MHS01/021

MATHS 102.

① $\sin^6 x$.

$$\int \sin^6 x \, dx$$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right).$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \cos^2 2x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + 1 + \cos 4x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \cdot 2\cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \cdot 2\cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right)$$

$$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{2} - \frac{\sin 6x}{6}\right] + c$$

$$= \frac{1}{192}\left[60x - 45\sin 2x + 9\sin 4x - \sin 6x\right] + c.$$

(2) ~~so~~ $\cos^4 x \sin^3 x.$

Rewrite the expression as.

$$\int \sin^3 x \cos^4 x \, dx$$

$$\sin^3 x \cos^4 x = (1-\cos^2 x)\cos^4 x \sin x \;\cancel{4}$$

$$= \cos^4 x \sin x - \cos^6 x \sin x.$$

$$\cancel{\int (\cos^4 x \sin x - \cos^6 x \sin x)\, dx.}$$

$$= \frac{-1}{5}\left(\cos^5 x + \frac{1}{7}\cos^7 x + c\right)$$

3) $\cos x \sin 3x.$

Rewrite as.

$$\int \sin^3 x \cos x \, dx.$$

Let $u = \sin x.$

$$\frac{du}{dx} = \cos dx$$

Therefore

$$\int \sin^3 x \cos dx.$$

$$\int u^3 du.$$

using reverse power rule

$$\int u^3 du = \frac{u^3+1}{3+1} + c$$

hence $u = \sin x$

$$\int \sin^3 x \cos x \, dx = \frac{\sin 4x}{4} + c$$