6)

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1) $\int \sin^6 x \, dx$.

Solu

$\int \sin^6 x \, dx = \int \sin^2 x \cdot \sin^4 x \, dx$

$= \int \frac{1-\cos 2x}{2} \cdot \left(\frac{1-\cos 2x}{2}\right)^2 \cdot dx$

$= \int \frac{1-\cos^2 x}{2} \cdot \frac{1 - 2\cos 2x + \cos^2 2x}{4} \cdot dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left[\frac{1+\cos 4x}{2}\right] - \cos 2x\left[1-\sin^2 2x\right] dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 2x \, dx$

$= \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x \, dx$

$\therefore \int \sin^6 x \, dx = \frac{1}{8}\left[\frac{5x}{2} - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + c$

2) $\int \cos^4 x \sin^3 x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x \, dx = \int u^4 \cdot \sin^2 x \cdot \cancel{\sin x} \cdot \frac{-du}{\sin x}$

$= -\int u^4 \cdot \sin^2 x$

$= -\int u^4 \cdot (1 - \cos^2 x) = -\int u^4 \cdot (1 - u^2)$

$= -\int u^4 - u^6 = \int u^6 - u^4$

$\int \cos^4 x \sin^3 x = \frac{u^7}{7} - \frac{u^5}{5} = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

3. $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \cdot \frac{u^3 \, du}{\cancel{\cos x}} = \int u^3 \, du$

$\int \cos x \sin^3 x \, dx = \frac{u^4}{4} + c = \frac{\sin^4 x}{4} + c$.

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