$\int \cos 2x \sin^2 2x\,dx = \frac{1}{2}\left(u^3/3 + c\right)$

$\int \cos 2x \sin^2 2x\,dx = \frac{1}{2}\left(\frac{\sin^3 2x}{3} + c\right)$

$\therefore \int \sin^6 x = 1/8\left[5x/2 - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$

2 $\cos^4 x \sin^3 x$

Solution

$\cos^4 x \sin^3 x = \int \cos^4 x \sin^2 x \sin x\,dx$

$\sin^2 x + \cos^2 x = 2$

$\therefore \sin^2 x = 1 - \cos^2 x$

$\cos^4 x \sin^3 x = \int \cos^4 x(1 - \cos^2 x)\sin x\,dx$

$= \text{Let } u = \cos x$

$du = -\sin x\,dx.$

$\cos^4 x \sin^3 x = -\int u^4(1 - u^2)\,du$

$\cos^4 x \sin^3 x = \int u^4 - u^6 \cdot du$

$\cos^4 x \sin^3 x = \frac{-u^5}{5} + \frac{u^7}{7} + c$

$\cos^4 x \sin^3 x = \frac{-\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

3) $\cos x \sin^3 x$

Solution

Let $u = \sin x$

$du/dx = \cos x$

$dx = du/\cos x$

$\int \cos x \sin^3 x\,dx = \int \cos x\, u^3 \cdot du/\cos x$

$\therefore \int \cos x \sin^3 x\,dx = \int u^3 du$

$\int \cos x \sin^3 x\,dx = u^4/4 + c$

$\therefore \int \cos x \sin^3 x\,dx = \frac{\sin^4 x}{4} + c$

Name: Imasuen Precious Imuehyaosa
DEPARTMENT: MBBS
MATRIC NO: 191mhs01206

1) $\sin^6 x$

Solution

$\int \sin^6 x\,dx$

$\int \sin^6 x = \int \sin^2 x \sin^4 x\,dx$

$\int \sin^6 x = \int \frac{1-\cos 2x}{2} \cdot \frac{1-2\cos 2x+\cos^2 2x}{4}\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x-\cos^3 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right)-\cos 2x(1-\sin^2 x)\,dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+\frac{3}{2}+\frac{3\cos 4x}{2}-\cos 2x+\cos 2x\sin^2 x\,dx$

$\int \sin^6 x = \frac{1}{8}\int \frac{5}{2}-4\cos 2x+\frac{3\cos 4x}{2}+\cos 2x\sin^2 2x\,dx$

$\int \sin^6 x = \frac{1}{8}\left[\frac{5}{2}x-\frac{4\sin 2x}{2}+\frac{3\sin 4x}{2}\times\frac{1}{4}+\int \cos 2x\sin^2 2x\,dx\right.$

$\int \cos 2x\sin^2 2x\,dx = \int \cos 2x\,(u)^2\,\frac{du}{2\cos 2x}$

$\boxed{\begin{array}{l}\text{let } u=\sin 2x\\ du/dx=2\cos 2x\\ \therefore dx=\frac{du}{2\cos 2x}\end{array}}$

$\int \cos 2x\sin^2 2x\,dx = \int u^2\,\frac{du}{2}$

$\int \cos 2x\sin^2 2x\,dx = \frac{1}{2}\int u^2\,du$

$\int \cos 2x\sin^2 2x\,dx = \frac{1}{2}\left(\frac{u^3}{3}+c\right)$