1) $\int \sin^6 x \, dx$

sol

$$\int \sin^6 x \, dx = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \cos^2 2x\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(2 - 4\cos 2x + 1 + \cos 4x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \times \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2} 2\cos 4x \times \cos 2x\right)$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \tfrac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{32}\left(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right)$$

$$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

Let $\sin^6 x = R$

$$R = \int (10 - 15\cos 2x + 6\cos 4x - \cos 6x]\, dx$$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{6} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

2 $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - u^2$

$\int \cos^4 x \sin^3 x = \int \cancel{\sin x} - \sin^2 x \, u^4 - \frac{du}{\cancel{\sin x}}$

$= -\int (1-u^2) u^4 du$

$= -\int u^4 - u^6 du$

$= -\left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

$\therefore \int \cos^4 x \sin^3 x =$

$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

3 $\int \cos x \sin^3 x \, dx$

Soln

$\int \cos x \sin^3 x \, dx$

$\sin x = \text{odd}$

$u = \sin x$

$\frac{du}{dx} = \cos x = 0$

$du = \cos x \, dx$

$= \int u^3 \times du$

$= \left(\frac{u^4}{4} + C\right)$

$\therefore \int \cos x \sin^3 x \, dx =$

$\frac{(\sin x)^4}{4} + C$