

MBB8

1) $\int \sin^6 x dx$

Also the same as $\int (\sin^2 x)^3 dx = \int \left[\frac{1}{2} (1 - \cos 2x) \right]^3 dx$

$\Rightarrow \frac{1}{8} \int (1 - \cos 2x)^3 dx = \frac{1}{8} \int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x) dx$

$\frac{1}{8} \left[\int (x - \frac{3}{2} \sin 2x) + \int 3 \cos^2 2x - \int \cos^3 2x dx \right]$

$\frac{1}{8} \left[\left(x - \frac{3}{2} \sin 2x + \frac{3}{2} x + \frac{3}{8} \sin 4x \right) - \int \cos^3 2x dx \right]$

To find the $\int \cos^3 2x dx$

$\int \cos^3 2x dx = \int \cos 2x (\cos^2 2x) dx = \int \cos 2x (1 - \sin^2 2x) dx$

let $u = \sin 2x$

$du = 2 \cos 2x dx$

$\frac{-du}{2} = \cos 2x dx$

$\int \cos 2x (1 - \sin^2 2x) = \int -\frac{du}{2} (1 - u^2) = \frac{1}{2} \int du (u^2 - 1)$

$\frac{1}{2} \left(\frac{u^3}{3} - u \right) = \frac{u^3}{6} - \frac{u}{2} = \frac{(\sin 2x)^3}{6} - \frac{\sin 2x}{2}$

$\therefore \int \sin^6 x dx = \frac{1}{8} \left[x - \frac{3}{2} \sin 2x + \frac{3x}{2} + \frac{3}{8} \sin 4x - \left(\frac{\sin^3 2x}{6} - \frac{\sin 2x}{2} \right) \right]$

$\int \sin^6 x dx = \frac{5x}{16} - \frac{3}{2} \sin 2x + \frac{3}{8} \sin 4x - \frac{1}{48} \sin^3 2x + \frac{1}{16} \sin 2x + C$

$\int \sin^6 x dx = \frac{5x}{16} - \frac{23}{16} \sin 2x + \frac{3}{8} \sin 4x - \frac{1}{48} \sin^3 2x + C$

$$2) \int \cos^4 x \sin^3 x \, dx$$

Since m is odd $u = \cos x$

$$\frac{du}{dx} = -\sin x, \quad dx = \frac{-du}{\sin x}$$

Also recall: $\sin^2 x = 1 - \cos^2 x$

$$\therefore \int \cos^4 x \sin^3 x \, dx = \int \cancel{u^4 \sin x} (1 - \cos^2 x) \frac{-du}{\sin x}$$

$$\therefore \int (u^2 - 1) \cdot u^4 \, du = \int (u^6 - u^4) \, du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$3) \int \cos x \sin^3 x \, dx$$

let $u = \sin x$, $\frac{du}{dx} = \cos x$

$$du = \cos x \, dx$$

$$\therefore \int \cos x \sin^3 x \, dx = \int u^3 \, du = \frac{u^4}{4} + C = \frac{(\sin x)^4}{4} + C$$

$$\therefore \int \cos x \sin^3 x \, dx = \frac{(\sin x)^4}{4} + C$$