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19/MHS01/258

Integrate the following functions:

(2) $\int \cos^4 x \sin^3 x \, d$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$\int \cos^4 x \sin^3 x \, dx = -\int u^4 \cdot \cancel{\sin^3 x} \cdot \sin^2 x \cdot \frac{du}{\sin x}$

$= -\int u^4 (1 - \cos^2 x) \, du$

$= -\int u^4 (1 - u^2) \, du$

$\cancel{= -\int u^4} \int (u^6 - u^4) \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$= \frac{(\cos x)^7}{7} + \frac{(\cos x)^5}{5} + c$

(3) $\cos x \sin^3 x$

let $u = \cancel{\sin x} \cos x$

$\frac{du}{dx} = \cancel{\cos x} - \sin x$

$\cancel{dx = \frac{du}{\cos x}}$ $dx = \frac{-du}{\sin x}$

$\int \cos x \sin^3 x = \cancel{\int \cos x \cdot u^3 \cdot \frac{du}{\cos x}}$

$= -\int u \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{du}{\sin x}$

$= -\int u (1 - \cos^2 x) \, du$

$$= -\int u(1-\cos^2 x)\,du$$
$$= -\int u(1-u^2)\,du$$
$$= \int (u^3 - u)\,du$$
$$= \frac{u^4}{4} - \frac{u^2}{2} + C$$
$$= \frac{(\cos x)^4}{4} - \frac{(\cos x)^2}{2} + C$$

① $\sin^6 x$

$$\int (\sin^2 x)(\sin^2 x)^2\,dx$$
$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)^2 dx$$
$$\cancel{\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 4x}{4}\right) dx}$$
$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right) dx$$
$$= \frac{1}{8}\int (1-\cos 2x)(1-2\cos 2x + \cos^2 2x)\,dx$$
$$= \frac{1}{8}\int 1 - 2\cos 2x + \cos^2 2x + 2\cos^2 2x - \cos^3 2x) \,dx$$
$$- \cos 2x$$
$$= \frac{1}{8}\int 1 - 2\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x - \cos^3 2x\,dx$$
$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x$$
$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left[\frac{1+\cos 4x}{2}\right] - (\cos 2x \cdot \cos^2 2x)\,dx$$
$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left[\frac{1+\cos 4x}{2}\right] - \cos 2x \cdot (1-\sin^2 2x)\,dx$$
$$= \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} + \cos 2x + \cos 2x \sin^2 2x$$

$$= \frac{1}{8} \int \left(\frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \cdot \sin^2 2x \right) dx$$

$$= \frac{1}{8} \left(\frac{5x}{2} + 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6} \right) + C$$